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à Chicoutimi

NONLINEAR PROGRAMMING FOR STOCHASTIC SHORT-TERM HYDROPOWER OPERATIONS PLANNING CONSIDERING UNCERTAIN PRICES

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RÉSUMÉ

L'hydroélectricité est l'une des principales sources d'énergie renouvelable et joue un rôle clé dans la production d'électricité au Québec et au Canada. Elle contribue à la durabilité énergétique, à la flexibilité du réseau et à la sécurité d'approvisionnement à long terme. Dans les marchés concurrentiels, la planification hydroélectrique à court terme est une tâche complexe en raison de l'incertitude des prix, de la dynamique non linéaire du système et de contraintes opérationnelles telles que les interactions entre réservoirs et les coûts de démarrage des turbines.

Cette thèse, présentée sous forme d'une compilation de trois articles, développe et analyse des modèles d'optimisation non linéaire pour améliorer la planification à court terme. Ces modèles intègrent les caractéristiques physiques des installations, les règles du marché et l'incertitude des prix. Pour représenter le comportement non linéaire des turbines sans modéliser chaque turbine individuellement, les trois articles utilisent des surfaces pré-calculées de puissance maximale reliant débit, volume du réservoir et production, réduisant la complexité tout en conservant les non-linéarités essentielles.

Le premier article introduit un modèle de programmation stochastique non linéaire en nombres entiers mixtes à deux étapes pour optimiser les stratégies d'enchères horaires sur le marché à J-1. Le modèle prend en compte la dynamique des réservoirs, les configurations de turbines et les contraintes du marché, et il est résolu à l'aide de méthodes exactes et d'un algorithme heuristique pour les cas de grande taille. L'évaluation utilise des données extraites de SHOP, un outil largement employé en Norvège pour la planification hydroélectrique à court terme.

Le deuxième article présente un modèle non linéaire de planification intégrant la couverture de la demande et les coûts de démarrage. Trois méthodes de résolution — une heuristique itérative, un algorithme génétique et une approche hybride — sont comparées à un modèle de référence optimisé, également à partir de données de SHOP.

Le troisième article propose un cadre d'optimisation en deux phases pour la soumission de blocs au marché de J-1. Dans la première phase, un modèle non linéaire déterministe génère un ensemble de profils de production faisables en tenant compte des coûts d'opportunité et des contraintes opérationnelles. Dans la seconde phase, un modèle linéaire stochastique à deux étapes sélectionne la meilleure combinaison de blocs selon plusieurs scénarios de prix. Le cadre est validé à l'aide d'une étude de cas réelle portant sur cinq centrales et six réservoirs dans le bassin de la rivière Orkla, en Norvège, et est comparé aux enchères horaires pour en mesurer l'efficacité.

Les résultats montrent que les modèles proposés capturent fidèlement le comportement non linéaire des systèmes hydroélectriques tout en maintenant des temps de calcul raisonnables, ce qui les rend adaptés à une mise en œuvre pratique.

ABSTRACT

Hydropower is one of the largest renewable energy sources globally and plays a key role in electricity production in Québec and Canada. It contributes to energy sustainability, grid flexibility, and long-term supply security. In competitive electricity markets, short-term hydropower scheduling is a complex task due to price uncertainty, nonlinear system dynamics, and operational constraints such as reservoir interactions and turbine startup costs.

This thesis, structured as a compilation of three scientific articles, develops and analyzes nonlinear optimization models to improve short-term hydropower planning. These models incorporate the physical characteristics of hydro systems, market rules, and price uncertainty. To capture nonlinear turbine behavior without explicitly modeling each turbine, all three articles use precomputed maximum power output surfaces that approximate the relationship between water discharge, reservoir volume, and generated power. This approach reduces model complexity while preserving essential nonlinearities.

The first article introduces a two-stage stochastic mixed-integer nonlinear programming (MINLP) model to optimize hourly bidding strategies in the day-ahead electricity market. The model accounts for reservoir dynamics, turbine configurations, and market constraints, and is solved using both exact methods and a heuristic algorithm tailored for larger-scale problems. It is evaluated using data extracted from SHOP, a widely used short-term scheduling tool for hydropower systems in Norway.

The second article presents a nonlinear short-term scheduling model that includes demand coverage and turbine startup costs. Three solution methods—an iterative heuristic, a genetic algorithm, and a hybrid approach—are compared and evaluated against an optimized reference model. The evaluation also uses data extracted from SHOP.

The third article proposes a two-phase optimization framework for block bidding in the day-ahead market. In the first phase, a deterministic nonlinear model generates a diverse set of feasible production profiles, considering opportunity costs and operational constraints. In the second phase, a two-stage stochastic linear model selects the best combination of blocks based on multiple price scenarios. The framework is validated using a real-world case study involving five power plants and six reservoirs in the Orkla River basin in central Norway. Additionally, the two-phase approach is compared with hourly bidding to evaluate its effectiveness.

The results show that the proposed models accurately capture the nonlinear behavior of hydro systems while maintaining reasonable computational times, making them well-suited for practical implementation in real-world operational environments.

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LIST OF ABBREVIATIONS

BP Bid Profile

DA Day-Ahead Market

DisCo Distribution Company

DP Dynamic programming

IPP Independent Power Producer

LP Linear Programming

LR Lagrangian Relaxation

MCP Market Clearing Price

MILP Mixed-Integer Linear Programming

MINLP Mixed-Integer Nonlinear Programming

NLP Nonlinear Programming

O.C. Opportunity Cost

SHOP Scheduling and Optimization Program for Hydropower

SP Stochastic Programming

STHS Short-Term Hydropower Scheduling

WPP Water Production Profile

DEDICATION

To my beloved mother, to the memory of my dear father, and to my beloved wife. With deepest gratitude and love.

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CHAPTER I

INTRODUCTION

Over the past decades, considerable efforts have been made to shift energy production toward cleaner and renewable sources, with the aim of ensuring sustainable energy generation. Hydropower is one of the largest sources of renewable energy and plays an important role in the electricity market. In these power plants, electrical energy can be stored in the form of water in the reservoir, offering multiple benefits such as flood control and operational flexibility. Due to their short start-up and shut-down times, hydropower units can quickly adjust their output in response to fluctuations in demand. Therefore, optimizing the operation of hydropower plants to maximize the use of water stored in the reservoir is important.

In recent years, as the world shifts toward cleaner and more sustainable energy sources, electricity markets have also evolved to support more competition and flexibility. These markets are usually divided into several stages, such as long-term contracts, the day-ahead market, intraday trading, and balancing markets.

Among these markets, the day-ahead market is the most important segment, as the majority of electricity transactions take place there. In this market, prices are formed based on the supply and demand submitted by market participants.

1.1 PROBLEM DEFINITION

In recent years, global climate change and the decarbonization of the world economy have prompted special attention to renewable energy sources as sustainable and environmentally friendly alternatives [2]. Hydropower is one of the most reliable, cleanest, cheapest, and largest sources of renewable energy in many countries, contributing more than 20% of global electricity generation [3, 4]. In Canada, hydropower accounts for over 60% of total electricity production, representing more than 80% of electricity generated from renewable sources [5]. Hydropower produces more than 90% of the electricity in Quebec and several other provinces, highlighting its critical role in Canada's electricity industry and energy supply [5].

This work focuses on short-term hydropower optimization for participation in electricity market bidding. Given that hydropower plants account for a significant share of electricity production in Norway, the Nordic electricity market, referring to the integrated and deregulated market comprising Norway, Sweden, Finland, and Denmark, which was established between 1991 and 2000 to promote competition and cross-border trading [6], is selected as the basis for modeling market rules and bidding structures. The approaches are applied to systems that reflect the characteristics and constraints of the Norwegian electricity sector, where hydropower plays a dominant role in energy production.

To ensure practical relevance, the proposed methods are tested on realistic systems, including one modeled using data from the Short-term Hydro Optimization Program (SHOP), with two hydropower plants and six turbines, and another configuration comprising seven reservoirs and six hydropower plants. These systems provide a representative basis for assessing the effectiveness and scalability of the developed approaches under real-world operational and market conditions.

Short-term hydropower optimization is a complex problem due to the presence of multiple interdependent factors, including the nonlinear nature of the production function, turbine efficiency, turbine start-up operations, and various uncertainties such as water inflows and market prices.

Power Production Function

The power production in the hydro system depends on water discharge, water head, and turbine efficiency [7]. Power output (W) in a single turbine is given by:

$$p(q,h) = g \cdot \eta(q) \cdot q \cdot h(Q,v) \cdot \rho_d, \tag{1.1}$$

where p is the power output (W), g is gravitational acceleration (m/s^2), ρ_d is water density (kg/m^3), η is turbine-generator efficiency, q is water discharge (m^3/s), and h is the net head (m), which depends on the total water discharge and the reservoir volume v (Mm^3) [8].

The net head is calculated as:

$$h(Q,v) = fb(v) - tl(Q) - pl(Q,q), \tag{1.2}$$

where $fb(v)$ is the forebay elevation, $tl(Q)$ is the tailrace elevation, and $pl(Q,q)$ represents penstock losses.

Turbine Efficiency

Turbine efficiency refers to the ratio between the mechanical energy extracted from water and the electrical energy actually produced. It depends on factors such as the net head and the turbined discharge, and typically varies between turbines. As each unit has its own efficiency curve, two turbines operating under the same hydraulic conditions may produce different levels of power.

Turbine start-ups

Frequent turbine start-ups not only reduce the lifespan of the units and increase maintenance costs, but also lead to energy losses during the transition to optimal operating conditions. Therefore, each start-up event is considered as a cost parameter in the optimization models.

Electricity Markets

Electricity is traded through several structured markets, including long-term contracts, the day-ahead market, intraday trading, and balancing markets. The day-ahead market, which is the focus of this research, operates on an hourly basis: participants submit their bids for the following day, indicating the quantity of electricity they are willing to sell or buy and at what price. Bids can take different forms. The simplest are *hourly bids*, where each hour is treated independently. More complex formats include *block bids*, which span several consecutive hours and must be accepted or rejected as a whole. An advanced variant, *profile block bids*, allows participants to submit blocks with variable hourly quantities, offering greater flexibility for systems with fluctuating production capabilities, such as hydropower [9]. Once all bids are collected, the market operator determines the hourly market-clearing price by matching aggregate supply and demand. This process introduces a key source of uncertainty, as final prices may vary significantly depending on demand, weather conditions, and bidding behavior [10]. In the electricity market, the producer is considered a price-taker, assuming that their energy bid cannot influence the market-clearing price [11].

Uncertainty

In hydropower systems can arise from several factors, including variable water inflows—mostly resulting from rainfall and snowmelt, equipment failures, and unpredictable changes in electricity demand. Price uncertainty, discussed earlier, is another major challenge that affects short-term scheduling and market participation.

In this thesis, price uncertainty is taken into account, while water inflows are assumed to be deterministic. Given that the Norwegian electricity market is one of the most competitive in the energy sector, its market structure is used as a reference for modeling price uncertainty.

Optimization problems in energy systems aim to maximize revenue or minimize costs, while considering real-world conditions and constraints [12]. In hydropower systems, such problems are particularly challenging due to their nonlinear and nonconvex nature, as well as the presence of many variables, large system sizes, and operational constraints. In addition, uncertainties such as inflow variability and market price fluctuations further complicate decision-making [13, 14]. Nevertheless, the significance of optimization lies in the fact that even small improvements can result in substantial economic and operational benefits over the planning horizon [15].

Short-term hydropower scheduling involves preparing operational plans to determine the optimal water flows through turbines, reservoir volumes, and minimizing turbine start-ups within a defined time horizon. The varying efficiency of turbines, combined with the nonconvexity and nonlinearity of production functions, significantly increases the complexity of this problem. In practical operations, scheduling must account for uncertain parameters such as inflows and electricity prices. In competitive electricity markets, prices are determined by supply and demand dynamics, resulting in price uncertainty for subsequent days. If these uncertainties are assumed known, the optimization model becomes deterministic, which fails to reflect real-world conditions.

Despite significant and valuable research on short-term hydropower optimization, most studies rely on linear models with deterministic assumptions. This is primarily due to the faster computational times offered by linear models, which make them attractive for practical use—especially in large-scale systems or real-time applications. In some cases, such simplifications are also justified by the physical characteristics of the system. For example, in small reservoirs, the variation in water volume may have a negligible effect on the water head, allowing the head to be treated as constant. To address the system's nonlinearities, researchers often apply linearization techniques or use piecewise linear approximations to model the production function. Although methods such as dynamic programming are better suited for capturing nonlinear behavior, they quickly become computationally infeasible as system size increases.

Beyond these nonlinear challenges, short-term hydropower scheduling is also affected by the uncertainty of electricity prices in competitive markets, where prices fluctuate based on supply and demand dynamics. Deterministic models are unable to properly reflect this uncertainty, which can expose producers to greater economic risks and lead to less effective bidding strategies. As a result, incorporating price uncertainty into optimization models is crucial for achieving realistic and efficient operational planning. These challenges highlight the necessity of developing optimization methods that not only handle nonlinearity and operational constraints but also take into account market uncertainties along with the specific rules and structure of electricity markets. Designing such approaches is essential for more accurate and practical decision-making in short-term hydropower scheduling.

1.2 OBJECTIVES

This thesis, structured as a compilation of scientific articles, aims to present and develop advanced nonlinear optimization models and solution approaches for short-term hydropower scheduling. The main goal is to address real-world operational constraints and uncertainties, particularly in competitive electricity markets, by combining practical modeling techniques with efficient solution methods. The research is divided into three independent but complementary studies, each targeting specific challenges in hydropower optimization.

The first objective is to improve hourly bidding strategies in the day-ahead electricity market by formulating a Mixed-Integer Nonlinear Programming (MINLP) model that incorporates market regulations, reservoir dynamics, and turbine operating constraints. A two-stage stochastic approach is used to capture electricity price uncertainty through multiple market scenarios. To solve the resulting complex model, both an exact method using the BONMIN solver and a heuristic method that iteratively fixes binary variables and solves the resulting nonlinear subproblems are applied. This work is tested using data from the Short-term Hydro Optimization Program (SHOP), and its performance is benchmarked against existing strategies.

The second objective focuses on developing efficient solution techniques for nonlinear short-term hydropower optimization problems involving operational constraints such as demand coverage and start-up costs. A detailed MINLP model is proposed using maximum power output surfaces to represent nonlinear relationships between reservoir volume, discharge, and power generation, while simplifying turbine combinations. Three algorithms are developed: a binary genetic algorithm, a heuristic method, and a hybrid strategy combining both. The goal is to balance computational efficiency with solution quality. These methods are validated through case studies based on SHOP data, demonstrating the hybrid approach's superiority in convergence speed and solution quality.

The third objective introduces a novel two-phase optimization framework for block bidding in the day-ahead market, explicitly considering price uncertainty. In the first phase, a deterministic MINLP model generates a large set of feasible block bids based on operational constraints, opportunity costs, and start-up costs. In the second phase, a two-stage stochastic linear programming model selects an optimal subset of blocks based on price scenarios, aiming to maximize expected profit. Relaxation techniques leveraging total unimodularity are used to enhance computational performance. This approach is validated through a real-world case study involving five power plants and six reservoirs located in the Orkla river basin in central Norway, specifically within the counties of Trøndelag and Møre og Romsdal.

Across all three studies, nonlinear models are favored due to the inherent nonconvexities and nonlinearities in hydropower systems. The methodology integrates exact solvers, heuristics, and hybrid methods to achieve practical and scalable solutions. This work contributes to both methodological advancements and real-world applicability in the context of short-term hydropower scheduling under uncertainty.

1.3 ORIGINALITY STATEMENT

The originality of this research lies in addressing key gaps in short-term hydropower optimization and bidding strategies through three main objectives. Most bidding models in the day-ahead market are linear. In this research, a stochastic two-stage nonlinear model was developed to optimize hourly bidding strategies in the day-ahead electricity market, fully considering operational constraints and price uncertainty. Additionally, an iterative heuristic method was proposed to efficiently solve the complex nonlinear problem, achieving high-quality solutions in a short time.

After clearing the market, producers must cover their commitments based on their bid offers submitted to the day-ahead market. At the same time, solving mixed-integer nonlinear short-term hydropower optimization problems becomes highly challenging when demand constraints and start-up costs are included. To address this, a mixed-integer nonlinear model was formulated. Given the complexity of solving such models with exact methods, three tailored approaches were introduced: a heuristic method, a genetic algorithm, and a hybrid heuristic-metaheuristic method. These methods focus on reducing complexity while allowing complex problems to be solved with acceptable accuracy and reduced computation time.

There is limited research on profile block bidding strategies, as most existing studies focus on standard block bids. This thesis presents a novel two-phase optimization framework specifically designed for profile block bidding in the day-ahead market. The proposed approach combines a deterministic nonlinear model in the first phase with a two-stage stochastic model in the second phase. This framework not only ensures solution accuracy but also achieves fast convergence, making it practical for real-world applications where quick decision-making is critical.

Each of these contributions introduces new methods and frameworks that improve both the accuracy and efficiency of short-term hydropower scheduling and bidding under realistic market and operational conditions.

1.4 PUBLICATIONS DERIVED FROM THE THESIS

Several scientific outputs have resulted from this doctoral research. Three technical reports have been published in *Les Cahiers du GERAD* and are listed in the author's official GERAD profile. In addition to these reports, the following peer-reviewed publications and conference proceedings were produced as part of this thesis:

- Jafari Aminabadi, M., Séguin, S., Fofana, I., Fleten, S.-E., Aasgård, E.K.
  Short-Term Hydropower Optimization in the Day-Ahead Market Using a Nonlinear Stochastic Programming Model.
  Energy Systems, 2023.

- Jafari Aminabadi, M., Séguin, S., Fofana, I.
  Hybrid Genetic Algorithms and Heuristics for Nonlinear Short-Term Hydropower Optimization: A Comparative Analysis.
  Procedia Computer Science, 246, 282–291, 2024. Elsevier.

- Jafari Aminabadi, M., Séguin, S., Fleten, S.-E., Aasgård, E.K.
  Optimizing Profile Block Bids in Short-Term Hydropower Scheduling: A Two-Phase Model for the Day-Ahead Market.
  Accepted and presented at the 11th International Conference on Control, Decision and Information Technologies (CoDIT 2025). To be published in the conference proceedings and submitted to IEEE Xplore.

1.5 THESIS OUTLINE

This section provides a brief overview of the chapters included in this thesis.

Chapter 2 presents the background and motivation for this research. It begins by introducing hydropower systems and their importance in energy production. Then, it discusses electricity market structures, focusing on market rules and bidding strategies. The chapter also reviews existing optimization models and highlights the main challenges related to nonlinearity, operational constraints, and uncertainty in short-term hydropower scheduling.

Chapter 3 describes the overall organization of the thesis and explains how the research is structured as a compilation of three scientific articles.

Chapter 4 is based on the first published article titled *"Short-term hydropower optimization in the day-ahead market using a nonlinear stochastic programming model"* [16]. This chapter develops a stochastic two-stage nonlinear model to optimize hourly bidding strategies in the day-ahead market. It also introduces an iterative heuristic method to efficiently solve the nonlinear problem.

Chapter 5 includes the second published article titled *"Hybrid Genetic Algorithms and Heuristics for Nonlinear Short-Term Hydropower Optimization: A Comparative Analysis"* [17]. This chapter focuses on nonlinear short-term hydropower optimization by incorporating demand constraints and start-up costs. It proposes three solution methods: a genetic algorithm, a heuristic approach, and a hybrid method combining both.

Chapter 6 is based on the third submitted article, which presents a new two-phase optimization framework for profile block bidding in the day-ahead market. The first phase generates feasible production blocks using a deterministic nonlinear model, while the second phase applies a two-stage stochastic model to select the best combination of blocks under price uncertainty.

Chapter 7 summarizes the main findings of this research, discussing how the proposed models and methods contribute to improving short-term hydropower optimization and bidding strategies.

Chapter 8 provides conclusions and suggests directions for future research, focusing on potential improvements and applications of the developed models in larger and more complex systems.

CHAPTER II

LITERATURE REVIEW

This chapter reviews the main studies related to short-term hydropower scheduling and the electricity markets in which these systems operate. The goal is to provide the technical and economic background necessary for understanding the modeling framework developed in this thesis.

The first part introduces the structure and components of hydropower systems and, building on the general overview provided in Chapter I, presents a more detailed discussion of short-term hydropower scheduling, including the objective function and key operational constraints.

The second part focuses on electricity markets, especially competitive ones. Since this work is centered on bidding strategies in competitive market environments—particularly the Norwegian electricity market—this section explains how these markets function, with an emphasis on market types, bidding mechanisms, and price formation in the day-ahead market.

The final parts of the chapter present the main solution methods found in the literature, covering both deterministic and stochastic approaches. In deterministic models, it is assumed that all future information, such as electricity prices and water inflows, is known in advance. Stochastic models, on the other hand, take into account uncertainty in future parameters by representing possible outcomes through different scenarios.

The concepts and methods discussed in this chapter provide the necessary background for understanding the modeling framework developed in the following chapters.

2.1 HYDROPOWER PLANTS AND THEIR MAIN COMPONENTS

Hydropower plants represent a fundamental pillar of renewable energy systems, utilizing the potential energy of water to generate electricity in an efficient and sustainable manner.

Depending on how they manage water resources, these plants are generally categorized into two major types: run-of-river systems and reservoir-based facilities.

Run-of-river hydropower plants exploit the natural flow and elevation gradient of rivers without the need for large-scale water storage. This approach, often regarded as cost-effective, circumvents the need for extensive dam construction, thereby reducing both capital expenditures and environmental disturbances such as habitat disruption and land submergence. Nevertheless, the electricity generation in such systems is inherently dependent on river discharge, making them susceptible to seasonal and climatic fluctuations.

In contrast, reservoir-based hydropower plants integrate a dam to store significant volumes of water, offering enhanced control over power generation. This capacity to store water enables operators to adjust output in response to fluctuating demand and market conditions, thereby improving system reliability and providing critical peak load management. However, these advantages come at the cost of more complex construction requirements and greater environmental challenges, particularly those related to land inundation and disruption of ecosystems.

A typical reservoir-based hydropower system comprises several interrelated components. The **reservoir** serves as the primary water storage unit, regulating availability and ensuring a stable supply for electricity generation. Water enters the system through an **intake** structure, which directs the flow into the **penstock**, a pressurized pipe that channels it toward the turbines. Within the **turbine**, the water's potential and kinetic energy is converted into mechanical energy, which is subsequently transformed into electrical energy by the **generator**. These core elements are housed within the **powerhouse**, and after energy conversion, the used water is released back into the river via the **tailrace** channel.

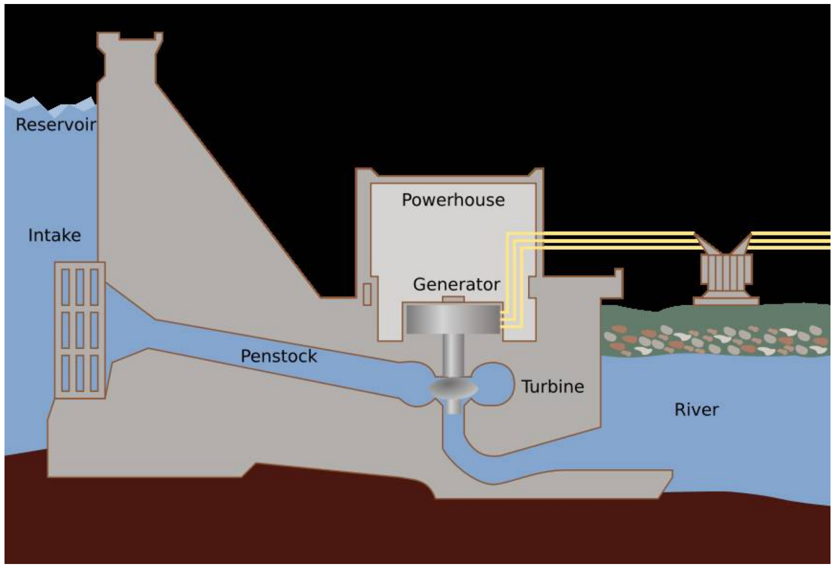

Figure 2.1 : Schematic of a reservoir-based hydropower plant. Adapted from [1]. Licensed under CC BY-SA 3.0 and GFDL.

A schematic representation of this process is illustrated in Figure 2.1, which highlights the spatial organization and functional integration of these components within a conventional reservoir-based hydropower plant.

2.2 HYDRO SCHEDULING PROBLEM

Due to the complexity of managing hydropower systems, various optimization processes are necessary. The hydro scheduling problem is commonly classified into three categories—long-term, mid-term, and short-term—based on the planning time horizon [18].

1. **Long-term:** Long-term models, which are typically stochastic, are used to determine optimal strategies for operating hydro systems over a multi-year period. These models are also valuable for market-oriented resource planning and price forecasting for power producers [19]. The long-term scheduling horizon generally spans from 1 to 5 years

and incorporates uncertainty related to inflows, electricity demand, and market prices through stochastic optimization and simulation approaches [18, 20].

2. **Mid-term:** Mid-term models serve as a bridge between long-term planning and short-term operations, ensuring consistent transfer of strategic decisions across different time scales [19]. The scheduling horizon typically ranges from 3 to 18 months and employs either stochastic or multi-scenario deterministic models [18]. Weekly scheduling at this level helps determine reservoir and plant-level water discharges [21, 22].

3. **Short-term:** Short-term hydro optimization involves two key problems: the unit commitment (UC) problem and the load dispatch problem. The UC problem focuses on determining the on/off status of generating units, while the dispatch problem aims to optimize water discharge through individual turbines [23]. The scheduling horizon for short-term planning is generally between 1 and 14 days, with a typical time resolution of one hour or less. Although often modeled deterministically [24], uncertain factors such as inflows and market prices may also be incorporated depending on system requirements [19]. These models often include detailed technical constraints such as start-up costs and delays, multiple turbines, multiple reservoirs with flow delays, minimum discharge limits, and minimum generation requirements [25].

2.3 SHORT-TERM HYDROPOWER OPTIMIZATION

Hydropower system scheduling problems can be modeled using either a plant-based approach [26, 27] or a unit-based approach [28, 29]. In plant-based models, the hydropower facility is typically treated as a single aggregated system, in which all generating units are assumed to share the same performance characteristics. Although this simplification can reduce modeling complexity, it may neglect operational differences among turbines that can significantly affect dispatch outcomes. However, in practical operations, individual turbines may have different efficiency profiles and dispatch constraints, which can significantly influence the quality of the scheduling results. To address these operational differences, a limited number of studies have proposed optimization models formulated at the unit level or through hybrid approaches that integrate both plant-level and unit-level modeling [23].

2.3.1 OBJECTIVE FUNCTION

The objective function in an optimization problem can focus on maximizing production [8, 30, 31] or revenue, especially in competitive electricity markets [32, 33, 28], minimizing operational costs [33, 28]. When the value of stored water in the reservoir is low or disregarded, the objective shifts towards reducing the number of unit start-ups and shut-downs in order to extend the lifespan of turbines and other equipment [34, 35]. In regulated markets with predefined load obligations and limited access to spot trading, minimizing the value of water used or spilled becomes a priority [36]. In such cases, the focus may turn to preserving water for future generation, especially when considering long-term potential [29].

2.3.2 CONSTRAINTS AND BOUNDS

The short-term hydropower scheduling problem is subject to various technical and operational constraints. Some of the key constraints are summarized below:

- **Water Balance Constraints:** These constraints ensure that the water volume stored in each reservoir is consistently updated over time. At each time step, the storage level equals the previous level plus the total inflows minus the total outflows. Inflows typically include natural inflows and delayed discharges from upstream reservoirs. In cascaded systems, a time delay—known as river routing delay—must be considered before upstream releases reach downstream reservoirs. Outflows consist of turbined water (used for electricity generation), bypass flow, and spillage. While turbined and bypass flows are controllable, spillage occurs when the reservoir reaches its maximum level and excess water is released without generating electricity. These dynamics are especially important in interconnected systems and must be accurately represented in the model [8, 30, 31, 16, 37, 38].

- **Storage and Spillage Limits:** Reservoir volumes must stay within minimum and maximum limits. Controlled spillage is restricted to avoid water loss, often with penalty costs for unregulated spills [8, 30, 16, 36, 39].

- **Head Variation and Hydraulic Losses:** Net head depends on reservoir levels and flow-related losses in tunnels, penstocks, and tailraces [28, 40, 41, 42].

- **Hydropower Production Function:** The nonlinear relationship between discharge, head, and power generation is often simplified using fixed efficiencies or piecewise linear approximations to reduce computational complexity [43, 44, 45, 46, 37]. Turbine efficiency specifically affects power production, as each turbine has its own efficiency, leading to different power outputs under identical water head and discharge conditions [15]. To further address nonlinearity and avoid handling individual turbine operations, maximum power output surfaces can be utilized. These surfaces capture the nonlinear relationship between reservoir volume, water discharge, and power production for different turbine combinations, thereby reducing model complexity while accurately representing system behavior [31, 15, 16].

- **Unit Operational Limits:** Each unit operates within defined power and discharge ranges, considering head-dependent limits and forbidden zones due to mechanical constraints [40, 15, 16, 35, 47].

- **Unit Commitment Constraints:** Binary variables represent unit status, with constraints to minimize frequent start-ups due to associated costs [31, 15, 41, 45].
- **Power Balance:** Generated power must meet market commitments or internal load demands, especially in deregulated markets [41, 34, 17, 37].

2.4 ELECTRICITY MARKETS

The electricity industry serves as a critical infrastructure upon which numerous sectors, including manufacturing and agriculture, heavily rely. In many countries, efforts to enhance efficiency, reduce costs, attract investment, and improve service quality have led to significant changes in the structure of electricity markets, a process known as restructuring. The deregulation of electricity markets began in the 1980s, notably in Chile and the United Kingdom [48, 49]. In traditional market structures, a centralized authority is responsible for coordinating operations with the primary objective of minimizing costs. By contrast, in restructured markets, various stakeholders—such as producers, retailers, and consumers—interact directly within a competitive environment, each aiming to maximize their own profit through market mechanisms [50].

The restructuring of the Norwegian electricity market began in the early 1990s, transitioning from a centrally coordinated monopoly to a competitive, market-based system [51]. This reform introduced deregulation of generation, allowing independent producers to schedule their operations based on market prices rather than central dispatch. With the introduction of organized electricity markets, producers were given the opportunity to participate directly in energy trading [51]. The goal of these changes was to enhance economic efficiency and market transparency. Nord Pool, the power market in the Nordic area, was established during this period and has since expanded to include several sequential markets for energy trading [13].

These foundational reforms laid the groundwork for the development of integrated regional electricity markets across the Nordic and Baltic countries.

2.4.1 ELECTRICITY MARKET STRUCTURES

Electricity markets have evolved through four main structural models, each progressively reducing monopoly control [52]: vertically integrated utilities, single buyer, wholesale competition, and retail competition.

Vertically Integrated Utilities In this traditional model, a single company controls generation, transmission, and distribution. Consumers had no choice but to purchase electricity from monopolized providers throughout most of the twentieth century [52, 53].

Single Buyer This model introduced competition in generation by allowing independent power producers (IPPs) to sell electricity to a centralized purchasing agency. While generation is competitive, purchasing and resale remain monopolized. It is common in countries like Kuwait, Qatar, UAE, Saudi Arabia, and Algeria [52, 53, 54].

Wholesale Competition Here, large consumers and distributors buy electricity directly from generators in a competitive wholesale market, while small consumers remain tied to regulated distributors. This model operates in countries such as Brazil, Mexico, Argentina, and parts of the US [52, 53, 55].

Retail Competition In this fully deregulated model, all consumers can choose their electricity supplier. While it promotes competitive pricing, it requires advanced infrastructure for measurement and data management [52, 53].

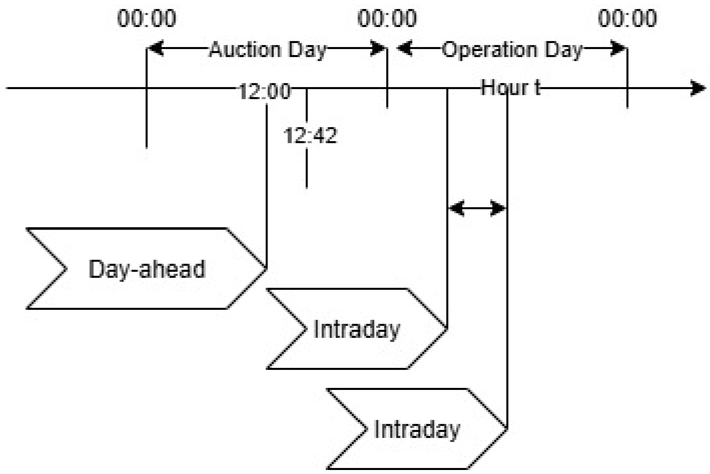

Figure 2.2 : Time-line for the clearing of the Nordic markets. Source: Adapted from Bringedal (2021).

2.4.2 SHORT-TERM ELECTRICITY MARKET

The short-term electricity market plays a crucial role in the Nordic power system, where trading is organized through a sequence of markets, namely the day-ahead, intraday, and balancing markets, as illustrated in Figure 2.2 [13, 56].

These markets are operated by Nord Pool, the common electricity market platform for Norway, Sweden, Finland, and Denmark. Nord Pool enables the coordination of supply and demand in an integrated regional framework that supports competitive price formation and efficient resource allocation [57].

In Norway, where more than 95% of electricity production is derived from hydropower, participation in the short-term markets is of particular importance [58].

The Norwegian electricity system is divided into five bidding zones (NO1 to NO5), each representing a separate price area. Local price differences may occur due to transmission constraints between zones, influencing the bidding strategies of producers [59]. These zonal structures, together with the flexibility of hydropower, make Norway a representative case of how renewable-dominated systems can interact efficiently with liberalized electricity markets [57].

The short-term electricity market provides a framework for producers to incorporate technical, hydrological, and economic aspects into operational planning. This becomes particularly important in hydro-based systems, where decision-making must account for uncertainty, especially uncertainty in electricity prices.

Day-ahead Market. The day-ahead market is the first and most important stage in the sequence of short-term electricity markets. Managed by Nord Pool, it enables market participants to submit price–quantity bids for each hour of the next day by noon (CET). These bids are used to construct aggregated supply and demand curves, and a market-clearing price is calculated for every hour using a blind auction process [60, 56].

This market handles over 80% of all physical electricity traded in the Nordic region [58]. Thanks to its transparency and liquidity, the day-ahead price serves as a benchmark for a wide range of financial contracts, including forwards and futures, which allow participants to hedge against price volatility [61, 60].

To support different production and consumption needs, Nord Pool offers several bid types: hourly bids (single hours), block bids (covering multiple hours), and flexible bids (which can be activated in any hour if the price condition is met). These formats give producers, especially hydropower operators, the flexibility to adjust their schedules efficiently [13].

For Norwegian producers, the ability to shift generation across hours plays a vital role in maximizing revenue and managing reservoir levels. Block and profile bids are commonly used to account for hydrological forecasts and operational constraints [62, 58].

Intraday Market. This market initiates its activity from closing the day-ahead market until an hour before the operation time, and it is an opportunity for producers to manage unfortunate commitments. Therefore, the producers can change the operation plan in the face of unforeseen events by purchasing and selling additional power [13]. In the day-ahead market, producers and consumers submit their offers before noon. The market operator calculates bid prices after receiving bids and offers from all market participants and then announces them publicly at around 1 p.m. each day [13, 56].

Balancing Market. The balancing market, also known as the real-time market, is where transactions and bidding are conducted close to the operating hour, typically around 45 minutes before delivery. This market is necessary for ramping flexibility and it organizes by the transmission system operator (TSO) as a single buyer [13].

2.5 DETERMINISTIC METHODS

In deterministic models, future information is assumed to be known and available in advance. Several solution methods have been developed for short-term hydropower optimization problems, generally categorized into two groups: exact methods and heuristic methods [63]. Several exact optimization methods have been developed to address the short-term hydropower scheduling (STHS) problem, each offering different trade-offs between modeling accuracy and computational complexity. These methods take advantage of the mathematical structure of the problem to find globally optimal or near-optimal solutions. The objective function varies depending on the application but typically aims to maximize energy production, minimize operating costs, or maximize revenue in competitive markets.

2.5.1 LINEAR PROGRAMMING (LP)

Linear Programming (LP) was among the earliest methods used for short-term hydropower scheduling, mainly due to the limited computing resources available at the time [37, 38]. In these early models, the optimization problem was formulated as a linear program and solved using commercial solvers, which are based on the classical simplex method or interior point methods. LP models are attractive because they are computationally efficient and can be solved to global optimality. However, they often simplify the problem by ignoring or linearizing nonlinear effects such as head variation and turbine efficiency curves [13]. As a result, while LP provides fast solutions, it may fail to accurately capture the physical characteristics of hydropower systems, leading to infeasible or suboptimal operating strategies in some cases [46].

2.5.2 MIXED-INTEGER LINEAR PROGRAMMING

Mixed-Integer Linear Programming (MILP) is a commonly used optimization method that handles problems where some variables must take integer values while others can be continuous [64]. In the context of short-term hydropower scheduling, MIP is particularly useful for representing discrete operational decisions such as unit start-up and shutdown, active units, and operational statuses [43, 14]. When the problem formulation remains linear, it leads to Mixed-Integer Linear Programming (MILP), which has been extensively used due to its balance between modeling flexibility and computational efficiency [65, 43]. MILP formulations allow hydropower scheduling models to handle discrete decisions while maintaining a structure that can be efficiently solved by commercial solvers like CPLEX and GUROBI [43, 44, 31].

Several studies have applied MILP to real-world hydropower systems. For instance, Daadaa et al. [31] proposed a MILP model that includes turbine efficiency curves and accounts for net head effects using an approximate correction factor. Their model, tested on two powerhouses in Canada, demonstrated noticeable improvements in electricity generation compared to historical operations. Similarly, in [65], a MILP model was developed for a system with seven power plants and 32 generating units, effectively handling water conversion and unit start-up/shutdown constraints, and was solved using CPLEX.

Other researchers have used MILP models to optimize different objectives. In [43], binary variables were introduced to model unit commitment decisions, while simplifying the hydropower production function through piecewise linear approximations. In [66], a MILP approach was designed to minimize the mismatch between generation and demand over a 24-hour horizon for a system with three hydro units. Similarly, [44] developed a MILP model aiming to maximize profit under forecasted electricity prices and inflows. Despite its advantages, MILP has some limitations. Linearizing nonlinear hydropower production functions can introduce deviations between the model results and actual system behavior [46, 13]. Moreover, as the number of binary variables increases, especially in large-scale systems, solving the MILP model can become computationally demanding. However, MILP remains one of the most widely adopted exact methods for hydropower scheduling, offering a practical compromise between the complexity of the modeling and the computational effort.

2.5.3 DYNAMIC PROGRAMMING

Dynamic programming (DP) was first introduced by Bellman in the 1950s [67]. It is a classical optimization approach that solves multistage decision problems by breaking them into smaller stages. At each stage, the best decision is made based on the current state and the expected cost of future stages. The basic recursive equation of DP is:

$$f_j^*(s_j) = \min_{x_j} \left\{ C(s_j, x_j) + f_{j+1}^*(s_{j+1}) \right\} \tag{2.1}$$

where j is the stage index, s_j is the system state at stage j, and x_j is the decision variable. $C(s_j, x_j)$ represents the immediate cost (or benefit) of decision x_j given state s_j, and $f_{j+1}^*(s_{j+1})$ is the optimal value function at the next stage. The next state s_{j+1} depends on both the current state and the decision taken.

In the context of short-term hydropower scheduling, the stages typically represent time periods (e.g., hours), the states correspond to reservoir volumes or the number of active units, and the decisions involve selecting turbine discharges or the on/off status of generating units. The objective can be to maximize revenue, minimize water usage, or reduce start-up and shut-down costs. For example, in [28], a nonlinear short-term scheduling model considering head variation and unit transitions was solved using DP for a real hydropower plant in Spain. Similarly, DP was applied to minimize start-up costs in a Brazilian reservoir with 18 turbines, although head variation was neglected to reduce complexity [40].

Despite its advantages in providing optimal solutions, DP suffers from the "curse of dimensionality," where the state space grows exponentially with the number of decision variables [68]. This makes it difficult to apply DP to large-scale systems, such as cascaded reservoirs or systems with many turbines [28, 68]. To overcome this, various strategies have been proposed, such as discretizing state variables efficiently or representing the system using aggregated states (e.g., total discharge or number of active turbines) instead of modeling each unit individually [40].

2.5.4 NONLINEAR PROGRAMMING (NLP)

Nonlinear programming (NLP) is mainly used in cases where a detailed and realistic representation of hydropower production is required, especially when handling nonlinearity and non-convexity [69, 70, 71]. In hydropower scheduling problems, nonlinearity arises from the dependence of turbine efficiency on both water discharge and reservoir head, where the head itself varies with the water level and discharge rate [13]. NLP models are particularly effective for solving unit load distribution (ULD) problems, where the objective is to allocate production optimally among a set of online units within a plant [23, 70]. By maintaining nonlinear formulations for both the objective function and constraints, these models better capture the physical behavior of hydropower systems compared to linear approximations.

Several deterministic NLP models have been proposed in the literature. For instance, Lu et al. (2004) presented a two-step iterative optimization method for reservoir management, where the first step solves an unconstrained optimization to determine reservoir limits, and the second step checks the feasibility of the solution [72]. Similarly, Seguin et al. (2015) proposed a two-phase approach to solve the short-term scheduling problem in a hydropower system, where a mixed-integer nonlinear programming (MINLP) model addresses the loading problem in the first phase, and the unit commitment problem is solved in the second phase [8].

It has been shown in [73] that nonlinear modeling can lead to higher revenues or improved operational performance, although it comes at the cost of increased computational time and model complexity. Furthermore, Seguin et al. (2015) demonstrated that, under specific conditions, the nonlinear problem with integer variables can be solved efficiently by relaxing the binary variables, significantly reducing the problem's complexity.

2.5.5 LAGRANGIAN RELAXATION (LR)

Lagrangian Relaxation (LR) is a popular decomposition method used to solve large-scale optimization problems by breaking them into smaller, more manageable subproblems [74, 39]. In short-term hydropower scheduling (STHS), LR is particularly effective when the system includes coupling constraints, such as water balance equations or power targets, that link multiple units across time or space [13, 39].

The key idea of LR is to relax the complicating constraints by incorporating them into the objective function with associated Lagrangian multipliers. This relaxation allows the original problem to be decomposed into independent subproblems, which can be solved more efficiently. Typically, the solution process involves three main steps: (1) dualization of the complicating constraints, (2) solving the resulting dual problem iteratively, and (3) recovering a feasible primal solution [23, 74].

Despite its advantages, LR faces challenges when the hydropower production functions are nonlinear and non-convex. Under such conditions, finding the true dual function becomes difficult, and convergence to an optimal solution cannot always be guaranteed [23].

2.5.6 META-HEURISTIC METHODS

Meta-heuristic algorithms can improve performance in large and complex hydro systems [75]. Common approaches include Ant Colony Optimization (ACO) [76], Particle Swarm Optimization (PSO) [77], Simulated Annealing (SA) [78], and Artificial Bee Colony (ABC) algorithms [79]. Additionally, hybrid algorithms have been developed to further enhance the efficiency of meta-heuristic methods [80, 81, 82]. Genetic Algorithms (GA), inspired by natural selection [83], are among the most widely used population-based meta-heuristics. GAs offer fast convergence, generate diverse solutions, and are capable of exploring the solution space efficiently [84, 85]. They use randomized operators such as selection, crossover, and mutation, and can be implemented as binary or real-coded algorithms [86]. Furthermore, GAs are often combined with other optimization methods to improve performance [84].

2.6 STOCHASTIC PROGRAMMING

Stochastic programming is a widely used approach for optimizing decisions under uncertainty. In the context of short-term hydropower scheduling, several sources of uncertainty must be considered to ensure realistic operation. These uncertainties include natural inflows—often driven by rainfall and snowmelt—fluctuating electricity prices in deregulated markets, variable power demand, and possible equipment failures such as unexpected turbine outages [87, 88, 89].

Unlike deterministic models, which assume perfect foresight of future parameters, stochastic models explicitly account for the variability of real-world conditions by considering multiple plausible future scenarios [90]. This allows system operators to make initial decisions while anticipating potential outcomes, and later adapt their strategies as uncertainty is resolved. By modeling these dynamics, stochastic optimization helps reduce operational risk and improve performance in energy markets [13].

2.6.1 SCENARIO TREES FOR HANDLING UNCERTAINTY

Scenario trees are commonly used in stochastic programming as an effective tool to handle uncertainty in decision-making problems. A scenario tree consists of nodes representing possible states of the system at discrete points in time, with branches connecting these nodes to illustrate transitions between different states as uncertainties unfold [90, 30]. Each path from the root node to a leaf node defines a distinct scenario representing one possible future realization of uncertain parameters, such as inflows or electricity prices [13].

In short-term hydropower optimization, scenario trees help operators incorporate uncertainties explicitly into their decision-making process. For example, at the initial stage, reservoir operation or market bidding decisions are made without precise knowledge of future inflows and market prices. As actual inflow or price information becomes available, decisions can be adapted accordingly in subsequent stages [88]. Thus, scenario trees enable a sequential decision-making framework, helping operators balance immediate operational decisions with future risks and opportunities.

However, the accuracy and computational complexity of stochastic models strongly depend on the scenario tree's size and structure. While a larger scenario tree typically provides better representation of uncertainty, it also results in increased computational burden. Conversely, a very limited number of scenarios may fail to capture critical variations in inflow patterns or market conditions. Therefore, choosing an appropriate number of scenarios and structuring the tree efficiently are essential tasks when using scenario-based optimization models [90, 60].

2.6.2 TWO-STAGE AND MULTI-STAGE STOCHASTIC MODELS

Depending on how decisions are made over time, stochastic models are divided into two-stage and multi-stage types [89].

In two-stage models, decisions are split into two steps: an initial decision made before the uncertainty is revealed and a second decision after the uncertainty materializes [91, 92]. Two-stage models are easier to solve because they involve one main decision before uncertainty is revealed and one adjustment after. They are well suited for problems like market bidding, where offers are made first and operations are adapted later once the market clears [13, 93].

In contrast to two-stage models, multi-stage stochastic models provide decision updates at several points as more information is revealed, making them more flexible but also significantly more complex. While two-stage models generally require handling a single probability distribution after the first period, multi-stage models involve a sequence of decisions across multiple periods. This sequential decision-making process allows the model to react dynamically to new information over time, enhancing adaptability in uncertain environments [10]. However, controlling the size of the tree is crucial to prevent exponential growth, often necessitating reductions at each stage using methods such as clustering [94, 15], copula-based discretization [95], or moment matching techniques [96].

Two-stage models are commonly employed in short-term hydropower bidding strategies focused on a single market, such as the day-ahead market. However, when producers participate across multiple markets (e.g., day-ahead, intraday, and balancing markets), multi-stage models become necessary to capture the sequential revelation of information about prices and commitments [60, 10].

2.6.3 MATHEMATICAL FORMULATIONS AND MODELING CHALLENGES

Different mathematical formulations have been applied to model stochastic hydropower scheduling, including Stochastic Mixed-Integer Linear Programming (SMILP) [88, 97, 60], Stochastic Successive Linear Programming (SSLP) [73, 10], and Stochastic Mixed-Integer Nonlinear Programming (SMINLP) [15]. These models must often address challenges such as nonlinear relations between production efficiency, water discharge, and head, as well as unit commitment decisions requiring integer variables [13, 43, 14].

Because solving such models can be difficult, many studies simplify the nonlinearities or use approximations [15, 98].

2.6.4 SHORT-TERM HYDRO OPTIMIZATION PROGRAM (SHOP)

The Short-term Hydro Optimization Program (SHOP) is a deterministic optimization model developed by SINTEF Energy Research to support short-term scheduling of hydropower systems. SHOP is widely used by Norwegian producers to plan daily operations and to prepare bids for the day-ahead electricity market. It can model a wide range of operational, physical, and market constraints in complex hydro systems [19, 99, 100].

SHOP is formulated as a mixed-integer linear programming (MILP) model, and its solution procedure consists of two main stages: unit commitment and unit load dispatch [42]. In the first stage, a MILP problem is solved to determine the on/off status of each generating unit in every time period. An estimated reservoir trajectory is used to guide the solution. Iterations are performed to stabilize water head variations, and reservoir volumes and head levels are updated after each iteration. Once the binary decisions are fixed and the unit commitment is solved, the second stage is triggered. In this stage, an LP model is used to compute the optimal generation levels based on the committed units, without changing the binary variables. The model typically operates on an hourly resolution over a time horizon of one to seven days. Inputs include forecasted inflows, market prices, and reservoir constraints. SHOP's detailed network structure represents river systems with reservoirs, turbines, bypasses, and spillways, and uses piecewise linear functions to approximate nonlinear relationships such as power output as a function of head and discharge [100].

Thanks to its modular structure, SHOP is highly adaptable and is used both in industrial applications and academic studies. Its stochastic extension, SHARM, incorporates uncertainty in inflows and prices through scenario-based modeling, enabling producers to evaluate trade-offs under risk [42].

2.7 SUMMARY

This chapter provided a comprehensive review of the main concepts, models, and market structures related to short-term hydropower scheduling. It began by introducing the technical components of hydropower systems and the classification of scheduling problems into long-term, mid-term, and short-term horizons. Special focus was placed on short-term models, including their objectives, constraints, and the distinction between plant-based and unit-based formulations.

The chapter then reviewed the structure of electricity markets, particularly the deregulated Nordic system and the day-ahead market operated by Nord Pool. Different market models—from vertically integrated utilities to full retail competition—were discussed, along with the roles of the day-ahead, intraday, and balancing markets in shaping operational decisions.

Next, the main solution approaches were categorized into deterministic and stochastic methods. Deterministic models, such as LP, MILP, NLP, and DP, were analyzed in terms of their accuracy, scalability, and computational performance. Stochastic models were introduced as a means of capturing uncertainty in inflows and prices, with particular attention given to two-stage and multi-stage formulations, scenario trees, and the trade-offs between realism and computational complexity. The role of SHOP as a widely used tool in the industry was also discussed.

Many studies have been carried out on short-term hydropower scheduling, leading to valuable contributions in the field. Nonlinearities are often either ignored, linearized, or approximated using piecewise methods—approaches that can lead to oversimplified models or increased computational complexity. Moreover, in competitive electricity markets such as the day-ahead market, failing to account for price uncertainty may result in unrealistic schedules and expose producers to significant financial risks. Therefore, there is still a clear need for models that can simultaneously capture nonlinear behavior, incorporate uncertainty, and handle operational constraints, while providing solutions within a short computation time. Addressing this gap is the main motivation of this thesis.

CHAPTER III

ORGANIZATION OF THE THESIS

This thesis, structured as a collection of scientific articles, focuses on the development of nonlinear optimization models for short-term hydropower scheduling under price uncertainty. The proposed models aim to reflect the physical and operational complexity of hydropower systems while maintaining computational efficiency and practical applicability. Each part of the thesis addresses a different aspect of the problem: the first article introduces a stochastic model for day-ahead bidding; the second article treats committed energy as a demand to be met, and compares three optimization methods for solving the resulting nonlinear problem; and the final article presents a novel two-phase framework for submitting profile block bids to the day-ahead electricity market.

In the first part, a two-stage stochastic nonlinear model is developed to optimize hourly bidding strategies in the day-ahead electricity market. The model integrates the nonlinear relationships between reservoir volume, water discharge, and power generation using pre-computed maximum power output surfaces. Electricity price uncertainty is modeled through discrete scenarios, and the optimization aims to maximize expected revenue while respecting system dynamics and operational constraints. The model is solved using both exact methods and a tailored heuristic approach that improves scalability by fixing binary variables in an iterative manner. The model is tested using the same input data as in SHOP, and the results are compared with SHOP outputs to evaluate the effectiveness of the proposed approach. This work is presented in Chapter 4 and has been accepted for publication in *Energy Systems*.

In the second part, we address the problem of short-term hydropower scheduling after the amount of committed production in the day-ahead market has been determined. This committed quantity is treated as a demand to be met, and a penalty is applied if the system fails to deliver it. A nonlinear mixed-integer model (MINLP) is formulated, including startup costs and nonlinear power generation functions. To solve this complex problem, we implement three methods: a binary genetic algorithm, an iterative heuristic, and a hybrid approach that combines both. Several test cases built from SHOP data are used to evaluate and compare the three solution methods in terms of result quality and computation time. For validation purposes, the results produced by the different approaches are compared with optimal solutions obtained from an exact solver. The results are presented in Chapter 5 and have been published in *Procedia Computer Science*.

The third part introduces a two-phase optimization framework for profile block bidding in the day-ahead market. In the first phase, a deterministic nonlinear model generates a wide set of feasible production profiles while considering operational constraints and opportunity costs. In the second phase, a two-stage stochastic linear programming model selects the optimal combination of blocks to maximize expected profits across a range of price scenarios. This approach addresses the limitations of traditional hourly bidding strategies by offering greater flexibility and robustness under uncertainty. A real-world case study involving six reservoirs and five power plants located in the Orkla River basin in central Norway is used to validate the model. The results are discussed in Chapter 6, and the corresponding article was presented at the 11th International Conference on Control, Decision and Information Technologies (CoDIT 2025) and will be included in the conference proceedings.

Finally, Chapter 7 concludes the thesis by summarizing the main findings of the three studies. It also highlights the strengths and limitations of the proposed methods and provides directions for future work, including the integration of long-term planning, the development of more efficient scenario generation techniques, and the application of the models to larger and more complex systems.

CHAPTER IV

SHORT-TERM HYDROPOWER OPTIMIZATION IN THE DAY-AHEAD MARKET USING A NONLINEAR STOCHASTIC PROGRAMMING MODEL

This article is published :

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4.1 ABSTRACT

Hydropower producers participate in the electricity market by providing bids in the day-ahead market auctions. Making good bids that obey all market rules and consider uncertain prices for large, interconnected hydropower watercourses is challenging. This investigation aims to find bidding strategies that attend to the market aspects and all constraints relevant to short-term hydropower production. This paper presents a stochastic mixed-integer nonlinear model and a nonlinear heuristic method for the bidding optimization problem and shows a comparison of the model's performance in two case studies. The comparison of the two models shows that their results are close and that the heuristic method can reach the optimal solution after a few iterations. The numerical experiments are also compared with results from the Short-term Hydro Optimization Program (SHOP), which is a software used for operational planning in the Nordic electricity market.

Keywords: Bidding, Short-term Hydropower Optimization, Nonlinear Programming, Day-ahead Market

NOMENCLATURE

Sets

t	Set of periods
c	Set of hydropower plants
S	Set of scenarios
I	Set of bid points
J_t^c	Set of active turbines (surfaces) in plant c at period t
r^c	Set of upstream hydropower plants of plant c

Parameters

w_t	Duration of period t (h)
ζ	Conversion factor from (m^3/s) to (Mm^3/h)
q_{min}^c	Minimum water discharge at plant c (m^3/s)
q_{max}^c	Maximum water discharge at plant c (m^3/s)
v_{min}^c	Minimum volume of reservoir c (Mm^3)
v_{max}^c	Maximum volume of reservoir c (Mm^3)
$v_{Initial}^c$	Initial volume of reservoir c (Mm^3)
v_{final}^c	Final volume of reservoir c (Mm^3)
P_i	Fixed price at bid point i
π^s	Probability of scenario s
ρ_t	Market price at period t
α_t	Reward factor for excess supply at period t
β_t	Penalty factor for shortage at period t

Variables

q_t^c	Water discharge at plant c and period t (m^3/s)
v_t^c	Reservoir volume at plant c and period t (Mm^3)
g_t^c	Water spillage at plant c and period t (m^3/s)
$z_{j,t}^c$	$\begin{cases} 1, & \text{if surface } j \text{ is chosen at period } t \text{ for plant } c \\ 0, & \text{otherwise} \end{cases}$
$\chi_{j,t}^c$	Power production for surface j at period t and plant c (MW)
yd_t	Committed hourly volume in the day-ahead market at period t
$xd_{i,t}$	Bid volume at period t and bid point i
H_t^s	Produced volume in scenario s at period t
zd_t^s	Supply shortage in scenario s at period t
zu_t^s	Oversupply in scenario s at period t

4.2 INTRODUCTION

Hydropower is one of the largest renewable energy sources and is one of the cheapest, cleanest, and most reliable sources of electricity production [3, 4]. Hydropower optimization is a nonlinear and non-convex problem. The large size of the system and the uncertainty of some important input parameters have made managing this system very complex [101], [102]. The deregulation of the electricity market and increased competition has led to the development of decision-making tools in the electricity industry [43]. In addition, due to maximizing profits in the electricity market and creating a balance between current and future profits, attending to uncertain parameters such as prices is necessary [87], [88]. The day-ahead market is an essential part of the electricity market because it has the largest share of the market [60]. Due to the nature of this problem, the focus of this paper is to find the optimal strategy for day-ahead market price-taker hydropower producers by using a mixed-integer nonlinear stochastic model and a heuristic method. The models with integer variables are too computationally demanding; therefore, a heuristic method also is used for this problem.

There is extensive research about optimal bidding; however, in the existing literature, the main focus is thermal generation. Hydropower plants usually have low start-up costs and are flexible in high ramping. For hydropower producers, optimal scheduling of water reservoir volume and releasing water is very important. Due to the expansion of wholesale electricity markets around the world, the importance of determining the optimal bidding in hydropower systems is increasing [13]. A multi-stage stochastic Mixed-Integer Linear Programming (MILP) model to optimize bids for 2 hours ahead market in Canada is presented in [103]. Operational constraints in the cascaded rivers and uncertain inflows and prices are considered in the model. One of the essential research on optimizing bidding is presented by Fleten et al. in the NordPool system [92]. The authors proposed a two-stage MILP in the day-ahead markets with uncertain prices. This model provides a price-dependent bidding curve, and the Nordic market rules and unit commitment decisions are included. In hydropower production aspects, the net water head is ignored. This model is extended in [97], and the coordinated bidding optimizing model is proposed by adding the intraday market. Another extension work of [92] is the multi-stage approach presented in [104], which considers the bid decisions in the day-ahead and intraday markets by integrating the short-term intraday with long-term inter-day decisions. The problem is defined as a Markov decision process and is solved by using approximate dual dynamic programming. To evaluate the stochastic bidding model in the interconnected river systems in [60], a stochastic MILP model without considering the effect of the water head is presented. In this model, both the bidding problem and the actual operational dispatch are modelled, and prices and inflows are considered uncertain. Uncertainty increases the complexity of the problem, and it may be too difficult to find a solution in MILP models, especially in large interconnected river systems; therefore, a stochastic linear model is formulated instead of the MINLP model in [105]. In this study, the effect of the linear approximation of start-ups on the quality of the results and the solution time is investigated. Another solution method in the unit commitment problem that the power producers in the Nordic market have widely used is Successive Linear Programming (SLP). In [73], an SLP model for operational stochastic short-term hydropower with uncertain future prices and inflow for the Nordic power industry is presented. This method has been implemented in SHOP, and the nonlinear head effect is modelled. In [10], the stochastic SLP model is used to optimize the power production in the hydropower bidding problem; a greedy algorithm is also presented to reduce the bid matrix. The water released is linked to power production by a piecewise linear concave production function for each generator. This study has shown that the problem size and computational time grow with the number of scenarios. The deterministic and stochastic models to obtain a bidding strategy are compared in [106]. Based on the results, the stochastic method has a better outcome for participants in the day-ahead market than deterministic models, as found in [103, 92, 60], and [73].

Nonlinear and nonconvex relationships are between decision variables such as water height, water discharge, and production efficiency in the hydropower problem. Dynamic programming can handle nonlinearity and nonconvexity, but in large problems, finding the solution is hard, which is called "the curse of dimensionality." Therefore, in most hydropower optimization problems, either the nonlinear effects are ignored or linear approximation methods reported in the literature [23, 13],

SHOP is an optimization model for scheduling hydroelectric power plants for daily operations that is provided as a software by SINTEF Energy Research. The model can handle various operational, physical and market constraints in complex hydro systems [19, 99]. The solution process in SHOP consists of two parts: unit commitment and unit load dispatch. The decision of turbines on/off in each period is determined by the unit commitment problem. The process is that the MILP problem with an estimation of the reservoir trajectories is solved. Then a number of iterations are performed to stabilize the head changes. The volume and water head of the reservoir are updated after each iteration, and this process continues until the stop conditions are met, and the unit commitment problem is solved. In the second part, the linear problem, unit load dispatch, is activated. The binary variables obtained in the unit commitment problem are fixed, and an LP model is used to obtain exact generation [42].

As reflected in the literature, mixed integer models have been widely used in the bidding optimization problem. In often studies, linearization and approximation techniques have been used to solve the problem. This paper presents a two-stage Mixed-Integer Nonlinear Programming (MINLP) for the price-taker producers in the day-ahead bidding based on the Nordic market. Instead of linearization and discretization, the maximum power output surface of the water discharge and volume of the reservoir for each turbine combination is used, leading to a nonlinear representation of the functions. This model considers the operational constraints of power production in the hydropower system and market rules. However, the existence of integer variables and their combination with stochastic programming raises the complexity of the problem. The contribution of this article is that an MINLP model is introduced to optimize bidding for the day-ahead market. Also, a heuristic method to solve the hydropower stochastic MINLP bidding problem by solving problems with less complexity in an iterative process is presented. The heuristic method can reach the right result in a short time. The two methods, more precisely an exact MINLP and the heuristic MINLP, are compared on datasets from the Nordic market and show that the average income in the MINLP model is slightly better in the examined cases, but the average solution time in the heuristic method is shorter. In some instances, the MINLP model could not find a solution.

4.3 SHORT-TERM HYDROPOWER SCHEDULING AND BIDDING PROBLEM

Short-term hydropower scheduling prepares the optimal strategy for daily operational plans. In order to achieve the desired performance, producers are looking to maximize revenues or minimize costs [107]. There are several methods for short-term hydropower optimization problems that can be divided into two general categories: exact methods and heuristic methods [63]. Also, the hydro system scheduling problems can be modelled in terms of plant-based [26, 27], or unit-based [28, 29]. The optimization problem in plant-based models is solved by aggregating at the plant level, which has the advantage of significantly reducing optimization complexity. In the unit-based models, the operational and physical conditions like turbine efficiency and limitations of the dispatch of the unit are considered [23]. MILP models [43, 65] are widely used, and piecewise linear approximations or other linearization techniques are employed to solve these models. Although they provide good results, the nature of the problem is nonlinear due to the relationship between turbine efficiency, water discharge, and water head [13, 14]. The combination of turbines in operation can be used, which causes that in addition to all the advantages of the unit-based conditions, it also reduces the complexity of the problem [31]. To participate in the Nordic day-ahead market, hydropower producers offer their bid matrix, a table containing the prices and power volumes per hour for the coming day to the market operator. This bid volume for each hour should not decrease as the price increases [10].

4.3.1 HYDROPOWER SYSTEM MODELING

As mentioned, hydropower optimization is generally a nonlinear problem, and it has operational constraints that should be considered in the model. This section introduces the power generation, the turbine's efficiency, the turbine's combination, and the operational constraints in the nonlinear model.

The power production in the hydro system depends on the water discharge and water head, and turbine efficiency [7]. Power output (W), in a single turbine is given as

$$p(q,h) = g * \eta(q) * q * h(Q,v) * \rho_d, \tag{4.1}$$

where p is the power output (W), g is the gravitational acceleration (m/s^2), ρ_d is the density of water (kg/m^3), η is the turbine-generator efficiency, q is the water discharge (m^3/s), h represents the net water head (m), which is calculated based on the total water discharge (sum of water discharge and spillage) and the volume of the reservoir v in (Mm^3) [8].

The gross water head is calculated from the difference between the tailrace elevation and forebay elevation. Further, the water friction in the penstock causes reduction in the water head, which is called penstock losses. Therefore, the net water head is calculated as shown in Eq. (4.2):

$$h(Q,v) = fb(v) - tl(Q) - pl(Q,q), \tag{4.2}$$

where fb is the forebay level of the reservoir unit, tl is the tailrace level of the reservoir unit, pl is the penstock losses of the unit (m). As mentioned, one of the most important factors in power production is turbine efficiency, and each turbine has its own efficiency. Turbine efficiency is important for power producers because the power production can be different for different units, even under similar conditions like the same net water head and water discharge, if the turbine efficiency is different. In addition to the total water discharge and water head, another factor that affects power generation in operational reality is the number of active turbines. Instead of working with turbines individually, a combination of turbines can be used. For example, suppose a power plant has a total of 4 turbines. In that case, the number of possible combinations for 1 active turbine is 4, 6 combinations for 2 turbines, 4 for 3 turbines, and 1 for 4 active turbines. In a hydropower system, if the number of turbines is large, the number of combinations will increase, making the problem more complex. Instead of working with all combinations, the maximum power output surface can be used for the number of turbines in each combination. The maximum output for each turbine combination is obtained by considering the water discharge and the volume of the reservoir. The first part of the two-phase model presented in [8] is used to obtain optimal power production. For the first phase, the loading problem, a MINLP is used to determine the power output, water discharge, reservoir volume, and the number of active turbines. The second phase is the unit commitment problem, where start-up costs are penalized. Since the second phase is executed after the loading problem, its results do not affect the loading problem, so the second phase is not considered in this problem. In the loading problem, the water discharge, q_t^c, and the volume of the reservoir, v_t^c, for each power plant C at the period T is limited by maximum and minimum levels, as in Eq. (4.3) and (4.4).

$$q_{min}^c \leq q_t^c \leq q_{max}^c \quad , \quad \forall t \in T, c \in C. \tag{4.3}$$

$$v_{min}^c \leq v_t^c \leq v_{max}^c \quad , \quad \forall t \in T, c \in C. \tag{4.4}$$

The maximum power output surfaces are obtained by polynomial equations and are used in the power production function equation, so the nonlinear relationship between water discharge and the volume of the reservoir is considered. The power production equations are:

$$H_t = \sum_{c \in C} \sum_{j \in J} \chi_{j,t}^c (q_t^c, v_t^c)\, z_{j,t}^c \quad , \quad \forall t \in T. \tag{4.5}$$

where $\chi_{j,t}^c(q_t^c, v_t^c)$ are the power output function for the surface j at period t, and plant c (MW), and binary variable $z_{j,t}^c$ has value 1 if the surface j is chosen at the period t. Eq. (4.6) ensures that only one surface is selected at each hour from the time horizon.

$$\sum_{j \in J} z_{j,t}^c = 1 \quad , \quad \forall t \in T, c \in C. \tag{4.6}$$

Water balance equations are given by:

$$v_{t+1}^{c} = v_{t}^{c} - \zeta\, w_t(q_t^c + g_t^c) + \zeta\, \delta_t + \sum_{r \in R} \zeta\, w_t(q_t^r + g_t^r) \quad , \quad \forall t \in T, c \in C. \tag{4.7}$$

Eq. (4.7) ensure the water balance of the power plants that are connected in series. The volume of the reservoir in the next hour, v_{t+1}^{c} is the volume of the reservoir at the period t, v_t^c minus the water discharge used for power production and spillage, $w_t(q_t^c + g_t^c)$, and ζ is the conversion factor from water discharge (m^3/s) to the reservoir volume (Mm^3/h), and plus inflow, δ_t, and water flow from upstream reservoirs into the reservoir, $w_t(q_t^r + g_t^r)$. The initial volume, v_1 and final volume, v_{final}, are considered input parameters to the model. It was shown in [8] and in the loading problem that integer variables could be relaxed if the unimodularity conditions in [64] are satisfied. This allows us to solve a non-linear problem continuously while maintaining an integer solution. The coefficient matrix is totally unimodular if these conditions are met:

1) All submatrices have elements that belong to the set {-1, 0, 1}.

2) Each column of the coefficient matrix contains at most two nonzero elements.

3) There exists a partition of the set of rows of the coefficient matrix, where each column containing two nonzero coefficients satisfies this partition.

Unimodularity conditions cannot be met when start-up costs or switching costs between maximum power output surfaces are considered in the objective function. Consequently, the switching costs between maximum power output surfaces are not taken into account in the model.

4.3.2 BID STRUCTURE

Hydropower producers offer their bids for the following day to the market operator in the day-ahead market, which is the main part of the European electricity market because most exchanges are done in these auctions. Producers and consumers submit their offer to the operator of the market organizer in the day-ahead market before noon. The marked operator calculates bid prices after receiving bids and offers from all market participants and then announces publicly at around 1 p.m. each day. Committed power is calculated by linear interpolation between each producer's bid curve and market price by the market operator. There are other markets, such as intraday and balancing to cover obligations, for the delivery of physical power. These markets increase flexibility and system stability [13], [56]. The balancing or real-time market is where the transaction and bidding are done near the operating hour, about 45 minutes or earlier. This market is necessary for ramping flexibility, and it is organized by the Transmission System Operator (TSO) as a single buyer[13].

The power producer participating in the day-ahead market can submit the hourly bid. As shown in Figure 4.1 the hourly bid includes the volume power offered for hour t, $XD_{i,t}$, and the price, $P_{i,t}$. The number of bid points, $I = \{\ 1, 2, ..., I\}$, are specified by market rules.

The user-determined set of fixed hourly prices, $P = \{\ p_1, p_2, ..., p_I\}$, is used to avoid nonlinear relation to determining bid volume and prices [92]. The hourly market prices are denoted by ρ_t and committed power volume at hour t, YD_t, is calculated with linear interpolation between the bid points on a bidding curve in Eq. (6.6).

$$YD_t = \frac{\rho_t - p_{i-1}}{p_i - p_{i-1}} XD_{i,t} + \frac{p_i - \rho_t}{p_i - p_{i-1}} XD_{i-1,t} \qquad if \quad p_{i-1} \leq \rho_t \leq p_i. \tag{4.8}$$

In the Nordic market, the bid by increasing prices must be non-decreasing, as shown in Eq.(9).

$$XD_{i,t} \geq XD_{i-1,t} \qquad t \in T,\, i \in I. \tag{4.9}$$

More details about the bidding curve and the problem of determining bids are presented in [108], [92], [91].

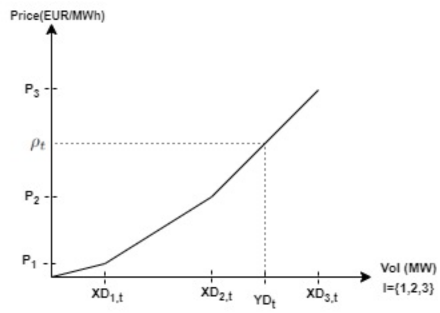

Figure 4.1 : Example of an hourly binding curve

4.4 METHODOLOGY AND OPTIMIZATION MODEL

In the day-ahead market, the prices are uncertain, so the model must consider this uncertainty to choose the optimal strategy. All the equations, including the production function and operating constraints, as well as the bidding structure presented in Section 4.3, are updated considering the uncertainty of the prices and a two-stage mixed integer nonlinear stochastic model is presented to optimize the bidding problem in Section 4.4.1.

Section 4.4.2 presents an iterative heuristic method to solve the bidding problem. The goal of this method is to find a suitable solution by reducing the complexity of the problem. In this method, the integer variables are fixed, and in each iteration, it attempts to improve the results by finding the appropriate value of integer variables.

4.4.1 TWO-STAGE MIXED INTEGER NONLINEAR STOCHASTIC MODEL

This two-stage nonlinear stochastic model allows determining the bid volume in the first-stage decisions at hour t, $xd_{i,t}$. Second-stage decisions are committed hourly volumes, $yd_{s,t}$, and these decisions depend on the price scenarios ρ_t^s, where the positive and negative imbalance between committed volume and power production are zd and zu. The objective function is to maximize the profit from offers, where π^s is the probability of each scenario, and α, β are the reward and penalty of participating in the balancing market. The MINLP stochastic model for the day-ahead market is given by the following model. Please note that the model is presented for a single hydropower plant, for the sake of clarity.

$$\max \sum_{s\in S} \pi^s \left(\sum_{t\in T} \rho_t^s\, yd_t^s + \sum_{t\in T} (\alpha_t\, \rho_t^s\, zu_t^s - \beta_t\, \rho_t^s\, zd_t^s)\right) \tag{4.10}$$

Subject to:

$$yd_t^s = \frac{\rho_t^s - p_{i-1}}{p_i - p_{i-1}} xd_{i,t} + \frac{p_i - \rho_t^s}{p_i - p_{i-1}} xd_{i-1,t} \qquad ,\text{if } p_i \le \rho_t \le p_{i+1}\,, \quad \forall t \in T,\, s \in S,\, i \in I \setminus \{1\}, \tag{4.11}$$

$$xd_{i,t} \ge xd_{i-1,t} \qquad ,\forall t \in T,\, i \in I \setminus \{1\}, \tag{4.12}$$

$$H_t^s = \sum_{j\in J} \chi_{j,t}^s (q_t^s, v_t^s)\, z_{j,t}^s \qquad ,\forall t \in T,\, s \in S, \tag{4.13}$$

$$\sum_{j\in J} z_{j,t}^s = 1 \qquad ,\forall t \in T,\, s \in S, \tag{4.14}$$

$$v_{t+1}^s = v_t^s - \zeta\, w_t (q_t^s + g_t^s) + \zeta\, \delta_t + \sum_{r\in R} \zeta\, w_t (q_t^{s,r} + g_t^{s,r}) \qquad ,\forall t \in T, s \in S, \tag{4.15}$$

$$q_{min} \le q_t^s \le q_{max} \qquad ,\forall t \in T,\, s \in S, \tag{4.16}$$

$$v_{min} \le v_t^s \le v_{max} \qquad ,\forall t \in T,\, s \in S, \tag{4.17}$$

$$v_1 = v_{Initial} \tag{4.18}$$

$$v_T \ge v_{final} \tag{4.19}$$

$$yd_t^s - H_t^s = zd_t^s - zu_t^s \qquad ,\forall t \in T, s \in S, \tag{4.20}$$

$$v_t^s \ge 0,\; q_t^s \ge 0 \qquad ,\forall t \in T, s \in S, \tag{4.21}$$

$$zu_t^s \ge 0,\; zd_t^s \ge 0 \qquad ,\forall t \in T, s \in S, \tag{4.22}$$

$$yd_t^s \geq 0,\ H_t^s \geq 0 \quad ,\forall t \in T, s \in S, \tag{4.23}$$

$$v_t^s, q_t^s, zu_t^s, zd_t^s, yd_t^s, H_t^s \in R, \tag{4.24}$$

$$z_{j,t}^s \in B. \tag{4.25}$$

Constraints (4.11) are the piecewise linear interpolation of the offer curve, which is the actual dispatch in scenario S and at hour t. Constraints (4.12) ensure that as the price increases, the bid curve is non-decreasing. Eq. (4.13) is the nonlinear production function for each price scenario at hour t, H_t^s, and is presented in Section 4.3.1. Constraints (4.14) limit the model to choose only one active turbine combination per hour t. The reservoir balance constraints are in Eq. (4.15), constraints (4.16) are water discharge bounds and constraints (4.17) are the limitation of reservoir storage level. The initial and final volume values are known as input parameters and specified in (4.18) - (4.19). Constraints (4.20) are the imbalance between the committed volume and the power production per hour, the shortage is bought from the intraday of the balancing market, and the extra energy is sold correspondingly.

4.4.2 NONLINEAR HEURISTIC BIDDING

Using the maximum output surface instead of all possible turbine combinations reduces the complexity of the problem. For example, all possible combinations for four turbines become 24 integer variables. Instead of these 24 variables, we use four maximum output surfaces for each number of active turbines. Despite the significant reduction in the combination of turbines, it is still a very complicated and time-consuming task to check all the active turbines within a time horizon of 24 hours or more, as well as with many scenarios. Solving the MINLP problem is associated with challenges; the computation time usually significantly increases in large-scale and complex problems. Therefore, a heuristic method is proposed that can reach a suitable solution with a low number of iterations. The main problem is broken into smaller problems and solved by an iterative approach. The problem-solving steps are shown in Figure 4.2.

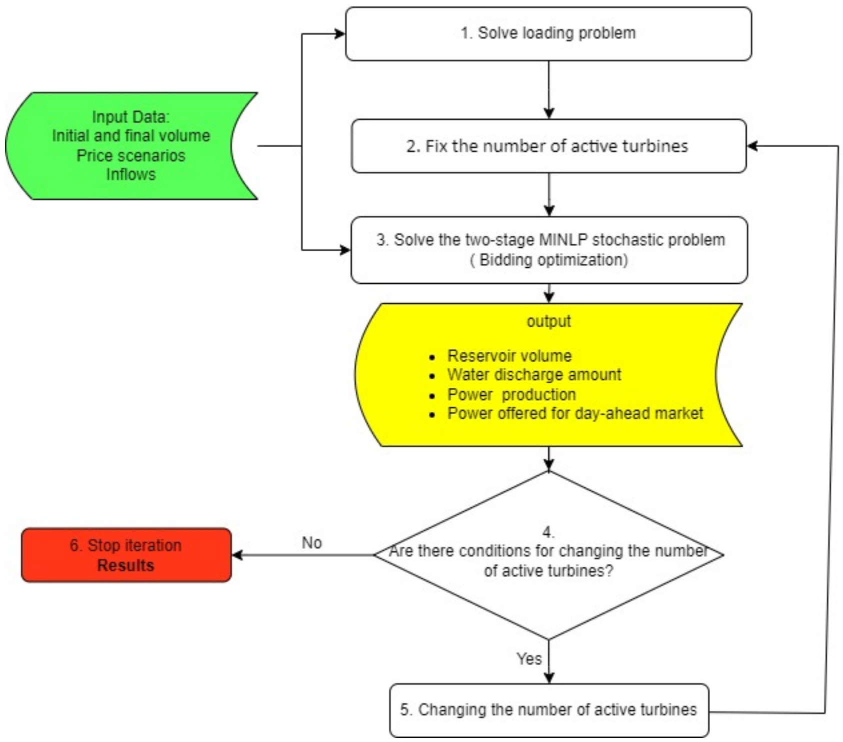

Figure 4.2 : Flowchart of a heuristic method for the bidding optimization for the day-ahead market

The loading problem is solved for all scenarios and problem inputs, including the initial and final volume and inflows, as shown in box 1. of Fig. 4.2. Eq. (4.26) is the objective function of the loading problem, which is to maximize the revenue in price scenarios. Equations (4.27) - (4.34) are the operational constraints of the hydropower problem.

$$\max \sum_{s\in S} \pi^s \sum_{t\in T} \sum_{j\in J} \rho_t^s \, \chi_{j,t}^s(q_t^s, v_t^s) \, z_{j,t}^s \tag{4.26}$$

Subject to:

$$v_{t+1}^{s} = v_t^s - \zeta\, w_t(q_t^s + g_t^s) + \zeta\, \delta_t + \sum_{r\in R} \zeta\, w_t(q_t^{s,r} + g_t^{s,r}) \quad , \quad \forall t \in T, s \in S, \tag{4.27}$$

$$\sum_{j\in J} z_{j,t}^s = 1 \quad , \quad \forall t \in T,\, s \in S, \tag{4.28}$$

$$v_0 = v_{Initial} \tag{4.29}$$

$$v_T \geq v_{final} \tag{4.30}$$

$$q_{min}^s \leq q_t^s \leq q_{max}^s \quad , \quad \forall t \in T,\, s \in S, \tag{4.31}$$

$$v_{min}^s \leq v_t^s \leq v_{max}^s \quad , \quad \forall t \in T,\, s \in S, \tag{4.32}$$

$$v_t^s \geq 0,\; q_t^s \geq 0 \quad , \quad \forall t \in T,\, s \in S, \tag{4.33}$$

$$z_{j,t}^s \in B \quad , \quad \forall t \in T, \in J, s \in S. \tag{4.34}$$

Since the coefficients of the constraints (4.28) - (4.30) are 0 and 1 and have only one element, the total unimodularity conditions in [64] are satisfied, so we can relax the integer variable and solve this problem for all scenarios. After solving the first step, the number of active turbines and the optimal power production in each scenario are determined. Then, we fix the integer variable, the number of active turbines in each scenario. The MINLP stochastic problem, equations (4.10) - (4.25), is solved with a fixed integer variable, as shown in box 3. of Fig. 4.2. Due to the absence of the integer variable in the nonlinear problem, the problem is solved as a continuous nonlinear problem. Fixing the integer variable may have introduced an error to the model, so the effect of this error can be reduced during an iterative process. For this purpose, it is reviewed whether the power production value will be increased by changing the number of active turbines, and the conditions mentioned in box 4. of Fig. 4.2 are as follows. After solving the bidding problem, the amount of power generation, the reservoir volume, and the water discharge in each scenario are known. On the other hand, there are maximum power output surface equations for each turbine combination, as explained in Section 4.3.1. The results obtained from the bidding problem can be replaced in the nonlinear surface equations as input. If there is a number of activities turbines that provides better power production with the same input, the number of active turbines will be changed. Suppose there are three turbines in the hydro plant, so we have three maximum surface equations. Based on the solution of step one in scenario S, the two active turbines are selected at time t, and the third step is solved with a fixed integer value.

$$z^{s}_{j,t} = z^{s}_{2,t} = 1 \quad , \quad \forall t \in T, s \in S \tag{4.35}$$

The results obtained in the third step include the power production, H^{*s}_{t}, water discharge, q^{*s}_{t}, and the reservoir volume v^{*s}_{t} at time t and scenario s.

$$H^{*s}_{t} = \chi^{s}_{2,t}(q^{*s}_{t}, v^{*s}_{t})\, z^{s}_{2,t} \quad , \quad \forall t \in T, s \in S \tag{4.36}$$

In the third step, we put the obtained results in the equations of other turbine combinations, i.e. one active turbine,H^{1s}_{t}, and three active turbines, H^{3s}_{t}.

$$H^{1s}_{t} = \chi^{s}_{1,t}(q^{*s}_{t}, v^{*s}_{t})\, z^{s}_{1,t} \quad , \quad \forall t \in T, s \in S \tag{4.37}$$

$$H^{3s}_{t} = \chi^{s}_{3,t}(q^{*s}_{t}, v^{*s}_{t})\, z^{s}_{3,t} \quad , \quad \forall t \in T, s \in S \tag{4.38}$$

If the results were better, the turbine combination is changed.

$$H^{1s}_{t}(q^{*s}_{t}, v^{*s}_{t}) \geq H^{*s}_{t}(q^{*s}_{t}, v^{*s}_{t}) \geq H^{3s}_{t}(q^{*s}_{t}, v^{*s}_{t}) \quad , \quad \forall t \in T, s \in S \tag{4.39}$$

It means that there is a number of active turbines that has a higher output with the same input than the previous condition. Therefore, instead of checking all turbine combinations, the number of active turbine changes only when the value of power production increases. Figure 4.3 shows the maximum output surface for three turbines, drawn in two dimensions for simplicity. As shown in the figure, after solving the stochastic model with a fixed integer variable of two turbines, the turbine combination will change in the next iteration if the amount of power production is in the marked parts. If the power production amount is in part A, the number of active turbines changes from two to one, and if it is in part B, the number of active turbines changes from two to three. In the next iteration, the number of changed turbines is fixed and the stochastic programming problem is resolved again. This iteration process continues until there is no change in the number of active turbines, after which the iteration stops.

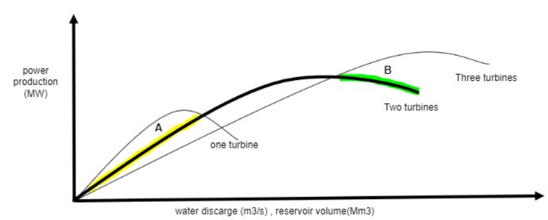

Figure 4.3 : Maximum output surface for three turbines

4.5 RESULTS AND DISCUSSION

The MINLP and heuristic method are tested on two cases with the data extracted from SHOP runs. This data includes power production values with different reservoir volumes and water discharge values for each turbine combination. Nonlinear equations of the maximum power output surface of each number of the active turbine are obtained using the extracted data and a polynomial approximation. The proposed method focuses on the day-ahead market, so the purchase amount in the balanced market is penalized, and selling the surplus amount is rewarded. Price scenarios, as shown in Figure 4.4, are selected to represent a typical pattern in the Nordic market, and inflows are regarded as deterministic. The summary of cases is shown in Table 4.1. All test cases are solved for a 24 hours planning horizon.

In each case, several instances with different input parameters, including the initial and final volume of the reservoir and the inflows, have been investigated. In each instance, to obtain the difference between the results of the objective function in the MINLP and the heuristic method, the difference is defined as:

$$\text{Difference}(\%) = (1 - \frac{\text{Objective function of Heuristic(EUR)}}{\text{Objective function of MINLP(EUR)}}) \times 100 \tag{4.40}$$

To evaluate the presented methods, the results of these two models have been compared with SHOP. In this program, which is based on successive linear programming and may also include mixed integer programming, a large number of cascaded water courses can be solved. The improvement percentage in the objective function of the MINLP and the heuristic method with SHOP is shown in equations (4.41) and (4.42).

$$\text{MINLP improvement}(\%) = (1 - \frac{\text{Objective function of SHOP (EUR)}}{\text{Objective function of MINLP (EUR)}}) \times 100 \tag{4.41}$$

$$\text{Heuristic improvement}(\%) = (1 - \frac{\text{Objective function of SHOP (EUR)}}{\text{Objective function of heuristic (EUR)}}) \times 100 \tag{4.42}$$

BONMIN [109] is an effective and efficient open-source solver used to solve MINLP and non-convex problems based on Cbc [110] and Ipopt [111] as building blocks. The stochastic MINLP and heuristic method are implemented in Julia, and the optimization software to solve stochastic MINLP is the BONMIN, and for the heuristic method, Ipopt is used. A laptop computer with an Intel Core i5 Processor and 8 GB of RAM is utilized to solve the models.

4.5.1 CASE A

Table 4.1 : A summary of case studies and methods evaluation

Case study	Number of instances	Number of reservoirs	Number of turbines
Case A	20	1	4
Case B	60	2	6
Validation results	9	2	6

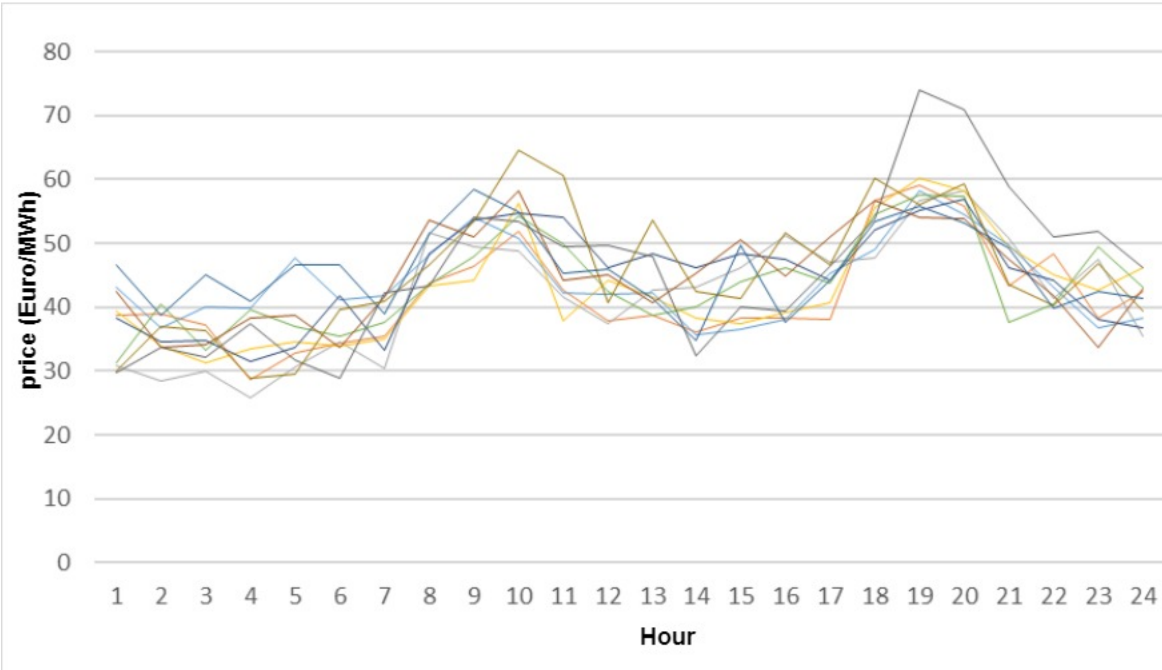

Figure 4.4 : Stochastic price scenarios for the day-ahead and 24-hour horizon.

This case study uses a power plant with four turbines and a reservoir. The maximum power generation capacity of this system is 345 MW, and the volume of the reservoir when full is 104.16 Mm^3. The total number of turbine combinations is 24, but for each number of active turbines, the power output surfaces are used, of which there are 4. A maximum power output surface should be selected per hour, so 4 binary variables are considered in the MINLP for each hour. 20 instances of various reservoir conditions, including full, half full, and almost empty, as well as 10 price scenarios and 24-hour time horizon, are examined. Inflows are considered deterministic. The comparison between the MINLP and the heuristic method is shown in Table 4.2. As mentioned in Section 4.4.1, the value of the objective function is revenue in euros for participation in the day-ahead market less the imbalance penalty. Eq. (4.40) is used to calculate the percentage difference between the objective function of the two methods. The number of active turbines selected in the first iteration will not change in the next iteration of the heuristic method if the initial and final volumes are such that 4 turbines, the maximum power production of the hydropower plant, are on for all scheduling periods. To make the comparison between the MINLP and the heuristic method more meaningful, the input parameters, such as the initial and final reservoir volume and the inflow, are set in such a way that there is not enough water to produce full in all hours.

Table 4.2 : Comparison revenue of the stochastic MINLP and heuristic method results in case A

Instance	MINLP (EUR)	Heuristic (EUR)	Difference (%)	Time MINLP (s)	Time HEU (s)	Diff Time (s)	Number of iterations
1	296,076	296,052	0.00%	13.10	25.55	-12.45	2
2	186,579	186,512	0.04%	114.29	51.12	63.18	4
3	324,377	324,381	-0.00%	22.28	23.10	-0.83	3
4	234,886	234,877	0.00%	31.75	30.88	0.87	3
5	178,121	178,043	0.04%	50.78	39.91	10.87	4
6	255,380	255,341	0.02%	50.75	43.39	7.36	4
7	244,565	244,520	0.02%	114.73	42.47	72.26	4
8	360,184	360,170	0.00%	19.48	34.47	-14.99	3
9	136,660	136,689	-0.02%	18.65	25.05	-6.39	3
10	218,479	218,359	0.05%	28.81	42.28	-13.47	4
11	138,268	138,246	0.02 %	36.55	32.53	4.02	4
12	212,724	212,675	0.02%	17.81	51.37	-33.56	5
13	282,276	282,291	-0.01%	18.64	49.04	-30.40	4
14	276,439	276,446	-0.00%	25.36	43.81	-18.46	4
15	170,818	170,776	0.02 %	15.77	43.28	-27.51	4
16	240,744	240,743	0.00 %	132.10	36.28	95.83	3
17	221,723	221,677	0.02 %	79.46	46.63	32.83	4
18	241,412	241,375	0.02 %	99.89	33.82	66.07	3
19	219,511	219,492	0.01%	76.17	46.03	30.14	4
20	202,485	202,465	0.01 %	18.54	45.39	-26.85	4

The model results in Table 4.2 show that both the MINLP and heuristic methods are efficient and can reach the solution in a short time. The average difference of the objective function value is 0.014% in favor of the MINLP. The average solution time for 20 instances in the MINLP is 51 seconds, and in the heuristic method is 40 seconds. Although the solvers of the MINLP and the heuristic method are different, the values of the objective functions are close, and there is not much difference between them. In the heuristic method, there is no guarantee of reaching the optimal solution, and because the solver of the two methods is not the same, some difference in the results is normal. In order to provide a more detailed comparison, hours 5, 11, 18 and 21 from instance 2 are selected. In this instance, the reservoir is considered half full, with high inflows. In Figure 4.5, the bid volume of the MINLP and heuristic method is shown for 10 scenarios. According to the results, bid volumes are similar in most scenarios.

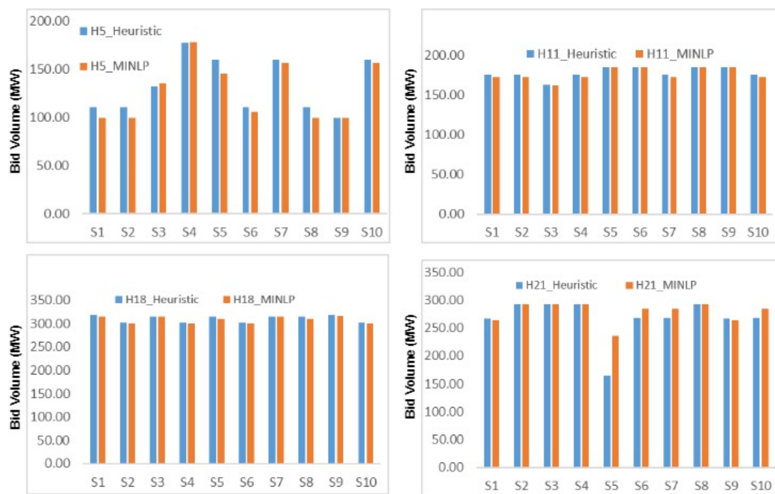

Figure 4.5 : Comparison of the bid volume per stochastic price for hours 5, 11, 18m and 21 and different scenarios in instance 2, case A

In Figure 4.6, the bid curves of the MINLP and the heuristic method in instance 2 are compared. Due to the closeness of the results between the two methods, it may be more appropriate to use the MINLP for small problems such as Case A due to the fact that the heuristic method does not guarantee an optimal solution.

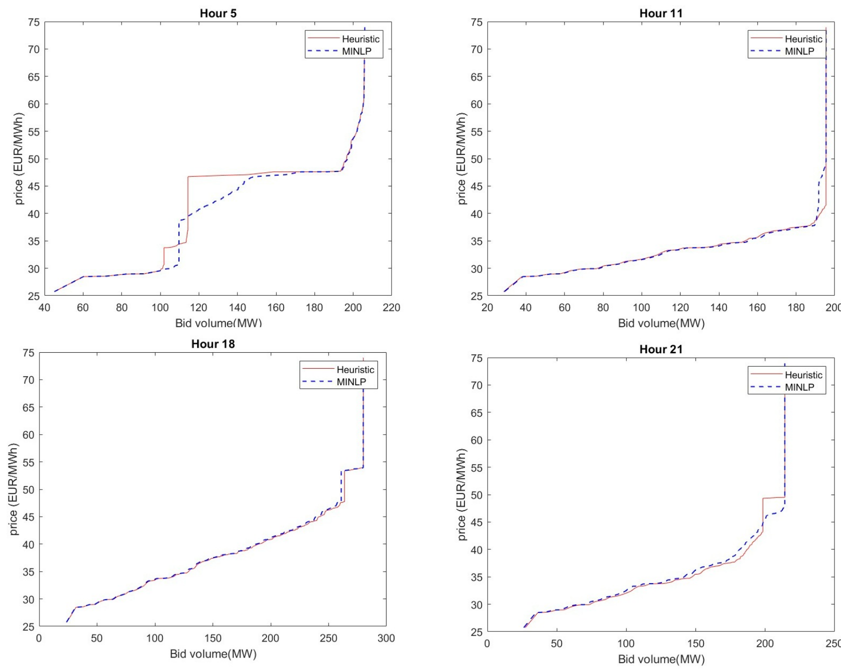

Figure 4.6 : Comparison of the bid curves at hours 5, 11, 18 and 21 for all scenarios in instance 2, case A

4.5.2 CASE B

In the second case study, two power plants are connected in series. The topology of the system is illustrated in Figure 4.7. The first power plant has two turbines, a maximum power generation capacity of 240 MW, and a maximum reservoir volume of 41.66 Mm^3. The second power plant has four turbines, a maximum power generation capacity of 345 MW, and a maximum reservoir volume of 104.16 Mm^3. The simulation is for a Norwegian system that participates in the day-ahead market with a 24-hour time horizon. The MINLP and heuristic method were tested on 60 instances for a planning horizon of 24 hours, with the initial and final volumes different, reservoirs almost empty, half-full and full. The inflows are considered deterministic and from low to high inflows. As shown in Table 4.3, rows 1 to 20 show results for low inflow, 21 to 40 for medium inflow, and 41 to 60 for high inflow.

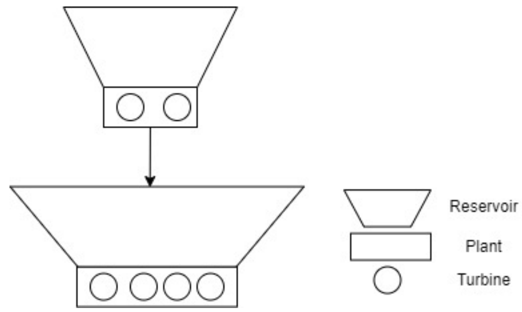

Figure 4.7 : Topology of the hydro system in case B

Table 4.3 reports the profit obtained in euros in each instance with the MINLP, the heuristic method, the solution time, and the difference between the two models. The results show that in 10 instances of the stochastic MINLP, no results were obtained with the BONMIN solver because the LP relaxation is infeasible or too expensive. However, the heuristic method provides the result after a few iterations. The solution time is too long to determine that the instance has no solution, so their time was not recorded in the table nor considered in the calculations. The average solution time for the nonlinear model is 131 seconds and for the heuristic method with an average of 4 iterations is 86 seconds. The MINLP has an average of 0.08% better results than the heuristic method. Figure 4.8 is a histogram of the difference percentage between the objective function in the MINLP and the heuristic method. As mentioned, the MINLP provides no result in 10 instances. So out of the remaining 50, in 27 instances, their percentage difference is between -0.05% and 0.05%; in 13 instances, the percentage difference is between -0.05% and 0.15%.

4.5.3 VALIDATION RESULTS WITH SHOP

To validate the MINLP and the heuristic method, the value of their objective function has been compared with SHOP. The objective functions and solution methods of these models

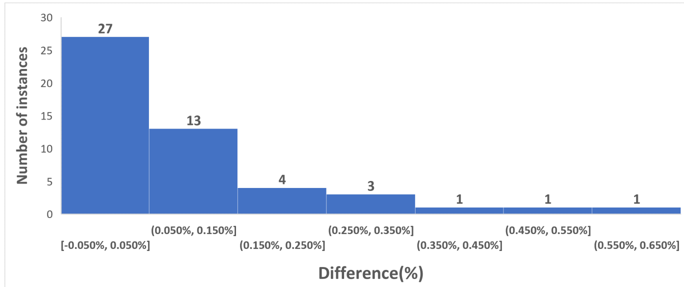

Figure 4.8 : Histogram of difference between stochastic MINLP and stochastic heuristic method, case B

are not the same; One of the differences is that in the MINLP and heuristic method, the power produced and the number of active turbines are determined in the first phase, the loading problem. In the second phase, the problem of unit commitment is solved, and the start-up costs are penalized; therefore, the start-up costs are not a decision variable in the bidding problem in the MINLP and heuristic method, and it is optimized in the second phase. But in SHOP model, the start-up costs are a part of the objective function. Another difference is that the water value is included in the objective function of SHOP, but in the MINLP and the heuristic method, the final volume is one of the input data. Therefore, for comparison, all conditions, such as the initial and final volume of the reservoir, are the same. We consider zero for the start-up cost and water value in all cases examined. There are also production constraints for turbines in SHOP, which were considered in the heuristic method and the MINLP for comparison. The inflow is considered into three categories: low, medium and high. The comparison results of the MINLP and heuristic method with SHOP are shown in Table 4.4.

The improvement percentage of the objective function value in the MINLP and the heuristic method compared to SHOP is obtained from equations (4.41) and (4.42), and it is displayed in Table 4.4 in the side column of the objective function of each model. As noted, because the formulation of the models is different from SHOP, the improvement of the objective function may not provide a complete picture of the benefits, and it is only used to validate the results. Figure 4.9 shows the comparison of the bid curves for instance 3 in the MINLP and the heuristic method with SHOP in hours 5, 9, 19, and 21. Due to the different models and their solvers, the results are different in some hours and very similar in others. Accordingly, the bid volume for the day ahead market is different at different prices in some hours based on the objective function improvement.

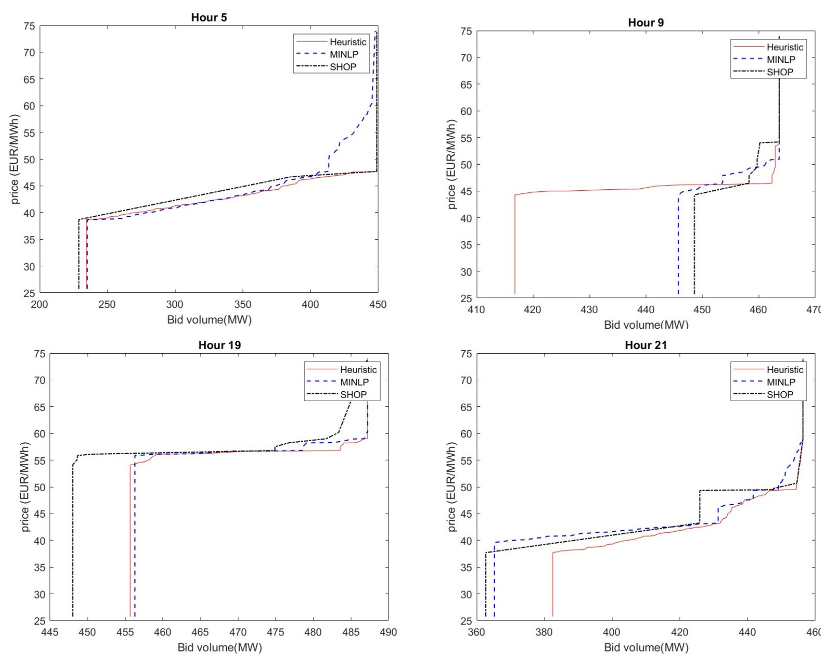

Figure 4.9 : Bid curves for a selected hour (Hours 5, 9, 19, 21)

4.6 CONCLUSION

This paper introduces a two-stage mixed integer nonlinear stochastic model to obtain efficient bids for a price-taking hydropower producer in the day-ahead market. Since hydropower optimization is generally a nonlinear problem, a MINLP was used to determine the optimal power generation in the bidding problem. In order to reduce the complexity of the problem, the maximum power output surface was used instead of working with all combinations of turbines. The results of the case studies show that the MINLP is efficient for small problems and can be solved in a short time. However, for larger problems, the solution time increased, and the binary variables and the complexity of the nonlinear problem make the MINLP unable to find the solution in some instances. Therefore, a heuristic method was proposed to solve the two-stage mixed integer nonlinear stochastic model, which can reach the appropriate solution after several iterations in a short time. Average revenues in case studies A and B are 0.014% and 0.08% higher, respectively, in MINLP than in the heuristic method. For future studies, another data set with more powerhouses in a larger system will be investigated, and uncertainty of inflows and prices will be considered simultaneously in the model. The presented methods can be compared with plant-based methods. Models can be developed by taking operational aspects into account, such as start-up costs and water value. The heuristic method can be combined with metaheuristic methods, and the results will be compared.

Table 4.3 : **Results for MINLP and heuristic methods for case B.**

Instance	Inflows	MINLP (EUR)	Heuristic (EUR)	Difference (%)	Time MINLP (s)	Time HEU (s)	Diff Time (s)	Number of iterations
1	Low	415,640	415,563	0.019%	146.03	72.40	73.63	4
2	Low	No result	525,654	-	-	54.96	-	3
3	Low	488,798	488,760	0.008%	160.38	74.95	85.43	4
4	Low	439,840	439,847	-0.002%	94.11	188.17	-94.06	5
5	Low	458,945	458,933	0.003%	229.52	220.56	8.96	5
6	Low	519,198	519,282	-0.016%	69.63	80.99	-11.36	4
7	Low	583,800	583,800	0.000%	15.61	31.87	-16.27	3
8	Low	382,630	382,642	-0.003%	173.33	64.34	108.99	4
9	Low	442,410	442,356	0.012%	18.32	66.39	-48.07	4
10	Low	411,106	410,746	0.088%	194.91	78.30	116.61	4
11	Low	432,330	432,409	-0.018%	123.93	87.46	36.47	5
12	Low	No result	458,164	-	-	68.55	-	4
13	Low	528,858	528,845	0.002%	36.43	50.25	-13.82	3
14	Low	506,363	506,398	-0.007%	62.90	64.37	-1.47	4
15	Low	463,316	463,207	0.024%	84.71	71.04	13.67	4
16	Low	495,952	495,652	0.060%	39.36	67.60	-28.24	4
17	Low	443,889	443,863	0.006%	299.39	98.47	200.93	4
18	Low	No result	482,266	-	-	117.64	-	4
19	Low	559,790	559,793	-0.001%	98.22	102.68	-4.46	4
20	Low	370,581	370,236	0.093%	149.08	97.89	51.18	5
21	Medium	440,527	440,524	0.001%	180.08	85.63	94.45	5
22	Medium	503,192	503,176	0.003%	27.37	85.83	-58.46	5
23	Medium	325,549	324,584	0.296%	95.65	106.05	-10.40	4
24	Medium	501,095	500,770	0.065%	396.96	110.18	286.78	4
25	Medium	557,098	557,083	0.003%	71.98	51.61	20.38	3
26	Medium	307,438	307,020	0.136%	106.44	126.43	-20.00	5
27	Medium	No result	372,167	-	-	95.76	-	5
28	Medium	453,719	453,868	-0.033%	204.04	92.29	111.75	5
29	Medium	314,211	314,292	-0.026%	152.83	43.83	108.99	4
30	Medium	429,609	429,519	0.021%	163.47	96.27	67.20	5
31	Medium	No result	356,977	-	-	107.10	-	6
32	Medium	411,493	410,729	0.186%	499.41	67.92	431.49	4
33	Medium	388,272	387,185	0.280%	104.42	70.31	34.10	4
34	Medium	433,909	434,010	-0.023%	94.48	56.30	38.18	5
35	Medium	571,885	571,872	0.002%	20.73	59.18	-38.45	2
36	Medium	No result	416,822	-	-	73.38	-	4
37	Medium	482,433	482,012	0.087%	90.46	82.68	7.78	4
38	Medium	418,097	417,490	0.145%	221.52	78.69	142.83	4
39	Medium	374,595	374,287	0.082%	57.20	197.34	-140.14	5
40	Medium	493,862	494,090	-0.046%	221.95	95.48	126.47	5
41	High	544,617	543,946	0.123%	32.99	69.76	-36.78	4
42	High	No result	337,140	-	-	97.11	-	6
43	High	398,340	397,600	0.186%	97.53	102.38	-4.85	6
44	High	383,772	383,187	0.152%	66.77	139.29	-72.52	4
45	High	No result	404,323	-	-	66.24	-	4
46	High	488,761	488,767	-0.001%	33.58	51.60	-18.02	3
47	High	No result	416,091	-	-	86.60	-	5
48	High	484,657	484,528	0.027%	182.25	70.42	111.83	4
49	High	406,413	406,616	-0.050%	50.83	66.80	-15.97	4
50	High	432,191	431,556	0.147%	133.11	77.25	55.86	4
51	High	404,549	402,236	0.572%	126.90	72.00	54.89	4
52	High	371,200	369,391	0.487%	144.67	66.16	78.51	4
53	High	401,446	400,178	0.316%	363.22	86.55	276.67	5
54	High	No result	377,770	-	-	46.51	-	4
55	High	407,531	407,105	0.105%	106.97	66.96	40.01	4
56	High	390,692	390,313	0.097%	85.52	82.09	3.42	5
57	High	447,669	447,465	0.046%	146.40	108.32	38.08	6
58	High	361,972	361,237	0.203%	92.69	67.71	24.98	4
59	High	322,351	321,070	0.397%	163.17	64.46	98.71	4
60	High	401,820	401,520	0.075%	35.95	87.43	-51.48	5

Table 4.4 : Validation results with SHOP in case B

Instance	Inflow	SHOP (EUR)	MINLP (EUR)	Improvement %	Heuristic (EUR)	Improvement %
1	Low	490,117	-	-	491,371	0.26%
2	low	288,609	-	-	291,975	1.17%
3	low	388,676	396,721	2.07%	396,603.0	2.04%
4	Medium	392,105	-	-	402,791	2.73%
5	Medium	518,067	-	-	518,067	0.00%
6	Medium	468,489	-	-	474,081	1.19%
7	High	513,283	513,283	0.00%	513,283	0.00%
8	High	477,594	485,393	1.63%	485,393	1.63%
9	High	320,898	-	-	324,437	1.10%

CHAPTER V

HYBRID GENETIC ALGORITHMS AND HEURISTICS FOR NONLINEAR SHORT-TERM HYDROPOWER OPTIMIZATION: A COMPARATIVE ANALYSIS

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5.1 ABSTRACT

In this paper, a Mixed Integer Nonlinear Programming (MINLP) for the short-term hydropower optimization problem considering operational constraints such as demand and start-up costs, is presented. Since solving the MINLP is complicated and, in many cases, impossible, three methods are proposed based on reducing the complexity, which is hybridized with the exact solver. Method A, a binary genetic algorithm; method B, an iterative heuristic method; and method C, using the iterative heuristic method in the genetic algorithm. Based on computational results in a case study, method B converges to a solution very quickly and with few iterations, whereas methods A and C perform more efficiently. A comparison between methods A and C indicates that method C not only reduces the computational burden for convergence but also yields better results. The proposed methods are evaluated by comparing them with optimal solutions. The results indicate that the proposed methods are highly effective in achieving favorable results.

Keywords: Nonlinear Short-term Hydropower optimization, mixed integer nonlinear programming, Genetic algorithm, Heuristic algorithm, Meta-heuristic algorithm

NOTATION

Sets

t	Set of planning periods
c	Set of hydropower plants
j	Set of active turbines (surfaces) in plant c at period t
r^c	Set of upstream hydropower plants of plant c

Parameters

ρ_t	Electricity price at period t
λ_t	Volume of demand in period t
α	Penalty factor for demand shortage
β	Reward factor for oversupply
w_t	Duration of period t (h)
ζ	Conversion factor from (m^3/s) to (Mm^3/h)
q^c_{min}	Minimum water discharge at plant c (m^3/s)
q^c_{max}	Maximum water discharge at plant c (m^3/s)
v^c_{min}	Minimum volume of reservoir c (Mm^3)
v^c_{max}	Maximum volume of reservoir c (Mm^3)
ε^c	Penalty factor for the start-up of turbines at plant c

Variables

q_t^c	Water discharge at plant c and period t (m^3/s)
v_t^c	Reservoir volume at plant c and period t (Mm^3)
g_t^c	Water spillage at plant c and period t (m^3/s)
δ_t	Inflow at period t (Mm^3)
$z_{j,t}^c$	$\begin{cases} 1, & \text{if surface } j \text{ is chosen at period } t \text{ for plant } c \\ 0, & \text{otherwise} \end{cases}$
$\chi_{j,t}^c$	Power production function for surface j at period t and plant c (MW)
Δ_t^c	Number of turbines turned on at period t and plant c
L_t	Supply shortage at period t (MW)
U_t	Oversupply at period t (MW)

5.2 INTRODUCTION

Hydropower is one of the most significant renewable energy sources used to produce electrical energy in the world and plays a decisive role in meeting global energy requirements [112]. Electricity producers aim to maximize revenue or minimize costs. However, the efficient management of hydropower systems presents intricate challenges due to the complexity of the system. Therefore, hydropower optimization processes are categorized into long-term, mid-term, and short-term problems [101, 107]. Long-term scheduling is typically based on stochastic models with uncertain variables to maximize future production potential [19]. In general, mid-term models have a one-year horizon and are used to manage the reservoir trajectories [19, 31]. Short-term hydropower models have planning times between one day and one week, considering operational constraints in order to determine the optimal daily production strategy. This planning mainly involves daily physical operations and is usually solved as a deterministic problem [107], although stochastic models have proven to be useful when there is variability in the inflows [30].

In view of the multi-dimensional relationships between variables such as water discharge, reservoir volume, and turbine efficiency, short-term hydropower production planning is naturally a nonlinear problem. The status (on/off) of turbines is determined by binary variables. Since it is difficult to work with nonlinear models with binary variables, either the production functions are linearized or the head effect is neglected [13]. The nonlinearity and nonconvexity of this problem, as well as the large dimensions of the problem, including a large number of decision variables, binary variables, and operational limitations, have made handling this system very challenging. Therefore, classical algorithms are not always sufficient to solve this problem, and other powerful methods are required [101, 113]. Various exact methods have been developed to optimize the hydropower problem, such as Linear Programming (LP) [38], Mixed Integer linear Programming (MILP) [31, 44], Mixed Integer Nonlinear Programming (MINLP) [8], Lagrangian Relaxation (LR) [74] and Dynamic Programming (DP) [40]. These algorithms can provide good results depending on the problem conditions, but they each have their own limitations. The optimal global solution can be obtained by linearizing the objective function and constraints in the LP model [14]. MILP has a high computational cost, especially if there are many turbines involved. Lagrange multipliers are difficult to find in LR, although it is a fast method. DP can handle the nonlinearity of the problem, but it suffers from the "curse of dimensions" as the dimensions of the problem increase [114]. Generally, the nonlinear effect is neglected or linearized due to the difficulty of working with nonlinear models, especially with integer variables [13]. A two-phase model is presented in [8], which provides another way to deal with integer variables and nonlinear aspects of the hydropower problem. The power output, water discharge, reservoir volume, and the number of active turbines are determined using a MINLP in the first phase, the loading problem. In the second phase, the start-up costs are penalized based on the unit commitment problem. In order to use this model, the unimodularity conditions in [64] must be satisfied. Meta-heuristic algorithms can improve performance in complex problems and in large hydro systems [75]. Thus, Ant Colony (AC) [76], Particle Swarm Optimization (PSO) [77], Simulated Annealing (SA) [78], and Artificial Bee Colony (ABC) [79] algorithms are used to optimize the hydropower problem. Additionally, hybrid algorithms such as [80, 81, 82] have been used to improve the performance and efficiency of meta-heuristic methods. Compared to classical methods, meta-heuristics and hybrid algorithms have higher flexibility in dealing with the complexity and limitations of the problem and can reach high-quality results at the right time. In a hybrid algorithm, the advantages of each algorithm are combined to improve the search space for the problem and speed up the convergence process.

The Genetic Algorithm (GA) is one of the most widely used algorithms based on population. It is inspired by natural selection mechanisms and has fast convergence and the ability to create a variety of solutions and search for the optimal result [84, 85]. Genetic algorithms are used for complex and nonlinear optimization of hydropower reservoir systems and multi-reservoirs in [115, 116, 86] and unit commitment problems [117].

The implementation of genetic algorithms is usually straightforward, and they can be easily hybridized with other optimization methods [84]. In [118], a hybrid Chaos optimization algorithm is used to improve GA performance and increase convergence speed. Using a hybrid algorithm of genetic algorithms and cellular automation to optimize reservoir operation problems, [119] demonstrates that the proposed algorithm is superior to genetic algorithms in achieving better results. Furthermore, modified genetic algorithms that increase the efficiency and speed of convergence of the algorithm have been developed for the hydropower optimization problem, including [120, 121, 122].

In this paper, a MINLP formulation is proposed to optimize the short-term hydropower problem, wherein instead of linearization or discretization, the maximum power output surface is employed to consider the nonlinear relationships between reservoir volume, water discharge, and power production for each number of active turbines based on [8]. Although the number of active turbines is used instead of working with all turbine combinations, the presence of binary variables, demand constraints, and start-up costs makes the exact solution of this model challenging and impossible in many cases. Moreover, if the number of active turbines, a binary variable, is known, the problem complexity is reduced, and exact solvers can be employed to solve the problem. Consequently, this paper presents three methods to obtain the number of active turbines, which are hybridized with an exact solver to optimize the short-term problem considering demand constraints and start-up costs. Therefore, in method A, the binary genetic algorithm is employed to solve the model since GA has a high degree of efficiency despite its ease of implementation and can be easily hybridized with other algorithms. In method B, an iterative heuristic method is employed to solve the problem and determine the number of active turbines. The method C is a new hybrid approach to solve the optimization problem in which the iterative heuristic method is applied to the GA.

5.3 MATHEMATICAL FORMULATION OF THE SHORT-TERM HYDRO-POWER PROBLEM

The purpose of the short-term hydropower optimization problem is to maximize revenue or energy generation within a time frame ranging from one day to one week. Various factors are discussed in this section regarding the short-term hydropower problem. The parameters in the power production function for a single turbine are gravitational acceleration g in m/s^2, the efficiency of the turbine η, water discharge q in m^3/s, the net water head h in m, which depend on the total water discharge Q (sum of water discharge and spillage) and volume of the reservoir v in (m^3). Additionally, ρ_d is the density of water (kg/m^3) considered constant [7]. Power output (W), in a single turbine is given as

$$p(q,h) = g * \eta(q) * q * h(Q,v) * \rho_d, \tag{5.1}$$

Table 5.1 : All combination of four turbines

1 active turbines	2 active turbines	3 active turbines	4 active turbines
1-2	12-23-34-13	123-124	1234
3-4	24-14	134-234	

The net water head is calculated by a function as shown in Eq. (5.2).

$$h(Q,v) = fb(v) - tl(Q) - pl(Q,q), \tag{5.2}$$

Where fb is the forebay elevation of the reservoir(m), tl is the tailrace elevation of the reservoir (m), and pl is the penstock losses of the unit (m). Turbine efficiency specifically affects power production, and since each turbine has its own efficiency, the turbines produce different power in the same conditions in terms of water head and water discharge. Operational power generation is affected not only by the total water discharge and water head but also by the number of active turbines. Combinations of turbines can be used instead of working with each turbine individually. As an example, Table 5.1 shows all turbine combinations for each number of active turbines for 4 turbines.

According to the Table 5.1, there are four combinations for one active turbine, six combinations for two active turbines, four combinations for three active turbines, and one combination for four active turbines. Many turbine combinations and integer variables increase the complexity of the problem. As shown in [8], instead of working with the individual turbines, the maximum power output surface can be used for each number of active turbines. On this surface, the maximum power production is obtained by considering the reservoir volume and the water discharge for each number of active turbines, so the nonlinear factors are also considered in the model. Instead of 18 combinations for 4 turbines, 4 maximum power surfaces are used. Besides determining the number of active turbines, other important concepts can be considered, including start-up, and coverage of demand. A large number of start-ups increase maintenance costs and reduce the turbine life cycle, so the start-up costs can be considered as another variable. The mathematical formulation of the short-term hydropower optimization problem is presented in this section. The purpose of the MINLP is to maximize revenue. The MINLP is given by:

$$\max \sum_{c\in C}\sum_{t\in T}\sum_{j\in J}\rho_t \times \chi^c_{j,t}(q^c_t, v^c_t) \times z^c_{j,t} \times \gamma_t + \sum_{t\in T}(\beta \times \rho_t \times U_t - \alpha \times \rho_t \times L_t) - \sum_{c\in C}\sum_{t\in T}\varepsilon^c \times \Delta^c_t \tag{5.3}$$

Subject to:

$$v^c_{t+1} = v^c_t - \zeta \times w_t \times (q^c_t + g^c_t) + \zeta \times \delta_t + \sum_{r\in R}\zeta \times w_t \times (q^r_t + g^r_t) \quad , \quad \forall t \in T, c \in C, \tag{5.4}$$

$$\sum_{j\in J} z^c_{j,t} = 1 \quad , \quad \forall t \in T,\, c \in C, \tag{5.5}$$

$$\sum_{c\in C}\sum_{t\in T}\sum_{j\in J}\chi^c_{j,t}(q^c_t, v^c_t) \times z^c_{j,t} \times \gamma_t - \lambda_t = U_t - L_t \quad , \quad \forall t \in T, \tag{5.6}$$

$$\Delta^c_t = j^c_t \times z^c_{j,t} - j^c_{t-1} \times z^c_{j,t-1} \quad , \quad \forall\, j \in J, c \in C, t \in T \setminus \{1\}, \tag{5.7}$$

$$v^c_1 = v^c_{Initial} \quad , \quad \forall\, c \in C, \tag{5.8}$$

$$v^c_T \geq v^c_{final} \quad , \quad \forall\, c \in C, \tag{5.9}$$

$$q^c_{min} \leq q^c_t \leq q^c_{max} \quad , \quad \forall t \in T,\, c \in C, \tag{5.10}$$

$$v^c_{min} \leq v^c_t \leq v^c_{max} \quad , \quad \forall t \in T,\, c \in C, \tag{5.11}$$

$$v^c_t \geq 0,\ q^c_t \geq 0 \quad , \quad \forall t \in T,\, c \in C, \tag{5.12}$$

$$z^c_{j,t} \in B \quad , \quad \forall t \in T,\, j \in J,\, c \in C. \tag{5.13}$$

The objective function in Eq.(5.3) includes four parts. The first is power production at each selected number of active turbines and at each hour, which is multiplied by prices. Since the power produced must cover the committed demand, in the second part of the objective function, the excess supply, U_t, is rewarded and the non-supply of demand, L_t, is penalized at each hour. β is the reward factor for oversupply, and α is the penalty factor for an undersupply of demand. Instead of working with all combinations of turbines, the maximum output surface is used, which reduces complexity and speeds up the solving process. In this method, it is not possible to determine exactly, which turbine is working, but the minimum start-up costs can be considered. Suppose that at hour t, the number of active turbines is 3 and at hour $t+1$ the number of active turbines is 4, so we know that at least one turbine has been turned on. Therefore, by considering the average start-up costs, ε^c, and the number of activated turbines at hour t, Δ_t^c the minimum start-up is penalized in the third part. The water balance constraints are in Eq.(5.4), and described by Eq.(5.5) limit the model to choose only one number of active turbine per hour t. Shown in Eq.(5.6) is the imbalance between demand volume and power production. Eq.(5.7) shows the number of turbines turned on per hour, which is achieved by switching between maximum power output surfaces. Eq.(5.8) specifies initial volumes. The objective function penalizes the lack of power production if it is less than the demand. Bounds on the variables are given in Eq.(5.10)-(5.11). Finally, Eq.(5.12) imposes nonnegativity and Eq.(5.13) defines binary variables.

5.4 METHODOLGY

The MINLP presented in Section 5.3 is used to formulate the short-term hydropower optimization problem in consideration of demand constraints and start-up costs. Despite the reduction in the dimensionality of the binary variables in the proposed model, solving this model with demand constraints and start-up costs remains challenging, and exact solvers are unable to solve it. Therefore, in this section, three methods are proposed to solve the model; all are based on reducing the problem complexity, determining binary variables, and solving the nonlinear model by fixing the binary variables using an exact solver.

5.4.1 BINARY GENETIC ALGORITHM (METHOD A)

Genetics and natural selection are the inspiration for the genetic algorithm [83]. Genetic algorithms are well-known algorithms that have been used in various optimization problems. Various operators are used to create a population, so the algorithm searches the problem space efficiently, and it can also be combined with other algorithms [84]. In this method, randomized operators such as selection, crossover, and mutation are used, and it is generally divided into two groups: binary GA and real GA [86].

Due to the difficulty in solving the MINLP, the binary GA is used to determine the number of active turbines at each hour of the planning horizon. By using the maximum output surface, there is no need to check all the combinations of the turbines, reducing the search space and increasing the speed of the algorithm. As shown in Figure 5.1, the number of active turbines is determined randomly by considering the condition that only one of the maximum power surfaces can be selected at each hour of the planning horizon. Afterwards, the number of active turbines, the integer variable, is fixed, and the nonlinear short-term hydropower problem is solved. Thus, all the equations presented in section 5.3 are taken into account, but instead of the integer variable $z^c_{j,t}$, the parameter $z^c_{Fixed,t}$ is fixed, and the nonlinear model is solved. In this method, since integer variables are fixed, a nonlinear model can solve the short-term hydropower problem continuously and quickly, and there is no need to estimate or assign fitness values to other variables. After evaluating the initial population, the parents' chromosomes are randomly selected, and the crossover operation is performed on the number of active turbines. For crossover, hybrid operators such as one-point, two-point, and uniform are used, and then the results are evaluated. In the next step, the mutation is done using a suitable strategy, single-point and multi-point mutation. As in the previous steps, the nonlinear model is continuously solved by fixing the number of turbines, and its results are evaluated. Based on the best result from the previous step, the population is updated, and this process continues until convergence conditions are met. It is considered the termination criterion of an algorithm if the objective function of NLP does not change after a number of successive iterations. In this method, instead of working with individual turbines, a combination of turbines is employed, which reduces the number of binary states and the number of iterations required to reach a solution. It is possible to obtain the objective function value and other variables by solving the nonlinear problem in each iteration by using the combination of genetic algorithms in the exact solution method.

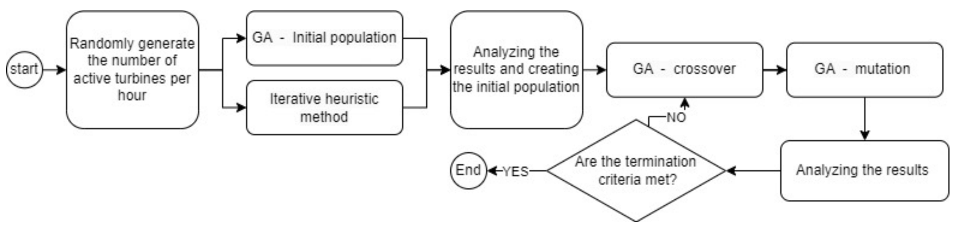

Figure 5.1 : Using the iterative heuristic method in the GA

The MINLP short-term hydropower problem can be simplified by using the maximum power output surface instead of working with all possible combinations of turbines, by fixing the integer variable, and by using GA to determine the number of active turbines. In the genetic algorithm, a systematic structure and iterations are employed to determine the number of active turbines per hour.

5.4.2 ITERATIVE HEURISTIC METHOD (METHOD B)

The genetic algorithm works randomly to determine the number of active turbines, and its results improve with some iterations. The purpose of this method is to determine the number of active turbines using a rule rather than random processes and to solve the hydropower problem more quickly and with fewer iterations than method *A*. As shown in Figure 5.2, the process starts with an initial estimate of the number of turbines, and like in Sections 5.4.1, the integer variable is fixed, and the nonlinear continuous model is solved. The number of turbines is updated according to the result and the heuristic method [16], which is explained in the following text.

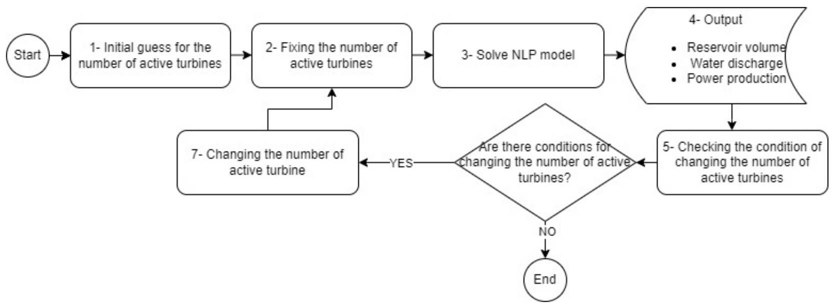

Figure 5.2 : Flowchart of a heuristic method (method *B*)

The output of the model, the reservoir volume, water discharge, and power generated at each hour, can be obtained after solving the nonlinear problem as shown in box 4 of Figure 5.2. The next step is to determine whether the change in the active turbines every hour increases the objective function's value. Therefore, the output obtained from the initial guess is used as an input to the maximum power output surface equations. The number of active turbines is altered if there is a number of active turbines that provide greater power production with the same input in the surface equation. This means that there are a number of active turbines that will produce more power with the same input, including water discharge and reservoir volume. Suppose there are four turbines in the hydro plant, so there are four maximum output surface equations as shown in Figure 5.3, drawn in two dimensions for simplicity. Also, three active turbines are considered as the initial guess at time t and the NLP model is solved with fixed integer values.

After solving the model with the fixed variable, its results are available every hour. This means the amount of water discharge at hour t, q_t^{*c}, the reservoir volume at hour t, v_t^{*c}, and

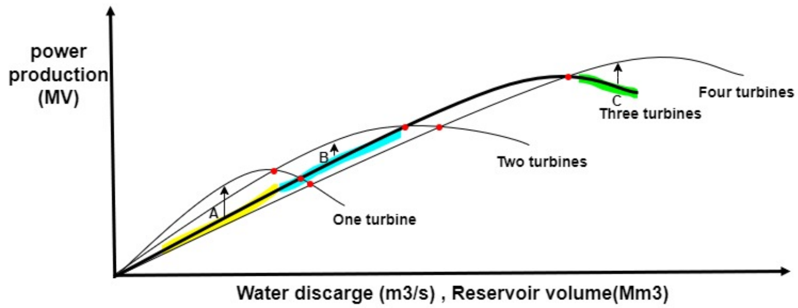

Figure 5.3 : Maximum power output surface for four turbines

power production at hour t are obtained from the following equation:

$$\chi^{c}_{3,t}(q^{*c}_{t}, v^{*c}_{t})\, z^{c}_{3,t}, \tag{5.14}$$

The power output values of one turbine $\chi^{c}_{1,t}(q^{*c}_{t}, v^{*c}_{t})$, two turbines $\chi^{c}_{2,t}(q^{*c}_{t}, v^{*c}_{t})$, and four turbines $\chi^{c}_{4,t}(q^{*c}_{t}, v^{*c}_{t})$ can be determined by putting the inputs into maximum power output surface equations. The number of active turbines at the time t changes from three to another, which has the highest increase in power production with the same input. At the time t, the number of active turbines does not change if no situation increases power production. Therefore, in Figure 5.3, the number of active turbines changes from three to one when the amount of production with three active turbines is in highlighted area A, and it changes to 2 when it is in highlighted area B, and finally, it changes to 4 when the amount of production with three turbines is in highlighted area C.

$$\begin{cases} \text{if } \chi^c_{1,t}(q^{*c}_t, v^{*c}_t) \geq \chi^c_{2,t}(q^{*c}_t, v^{*c}_t) \geq \chi^c_{3,t}(q^{*c}_t, v^{*c}_t) \geq \chi^c_{4,t}(q^{*c}_t, v^{*c}_t) & \Rightarrow \text{Area A} \\ z^{*,c}_t = z^c_{1,t} \\ \text{if } \chi^c_{2,t}(q^{*c}_t, v^{*c}_t) \geq \chi^c_{1,t}(q^{*c}_t, v^{*c}_t) \geq \chi^c_{3,t}(q^{*c}_t, v^{*c}_t) \geq \chi^c_{4,t}(q^{*c}_t, v^{*c}_t) & \Rightarrow \text{Area B} \\ z^{*,c}_t = z^c_{2,t} \\ \text{if } \chi^c_{4,t}(q^{*c}_t, v^{*c}_t) \geq \chi^c_{3,t}(q^{*c}_t, v^{*c}_t) \geq \chi^c_{2,t}(q^{*c}_t, v^{*c}_t) \geq \chi^c_{1,t}(q^{*c}_t, v^{*c}_t) & \Rightarrow \text{Area C} \\ z^{*,c}_t = z^c_{4,t} \end{cases} \tag{5.15}$$

Consequently, instead of randomly changing the number of turbines per hour from the planning horizon or checking other situations, this method is used to change the number of active turbines for each hour, and then the NLP model is solved with the applied changes, and this process continues until there is no change in the number of active turbines.

Despite having advantages such as the appropriate efficiency in finding the number of turbines per hour, this method may have problems and cannot work well at breaking points. This method can also be affected by the initial guess of the number of turbines. Therefore, a method that covers the search space of the problem well and reaches the right solution in a shorter number of iterations is required.

5.4.3 ITERATIVE HEURISTIC METHOD IN THE GA (METHOD C)

As mentioned, using the maximum power output surface instead of all turbine combinations can expedite solution-finding. The GA, despite its appropriate speed, which uses a random process at all steps, may not be effective for large, complex problems. The heuristic iterative method uses a fast algorithm instead of random methods to determine the number of active turbines. However, the efficiency of this method depends on the initial guess of the number of turbines and it also does not work well at breakpoints. Therefore, a method is introduced that benefits from the advantages of each method presented in the previous sections. The GA searches the problem area well, and the heuristic algorithm can approach the optimal solution at a suitable speed, so, as shown in Figure 6.1, the heuristic algorithm can be used inside the GA. To avoid limiting the search space in the genetic algorithm, only part of the of the initial population, derived from the heuristic algorithm results, is utilized. This approach is thus applied to the creation of the initial population in the GA. As discussed in Section 5.4.1, the entire process, including crossover and mutation, continues until a termination criteria is met.

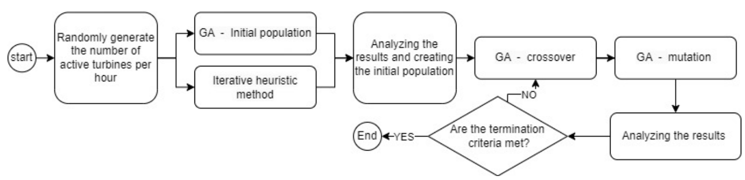

Figure 5.4 : Using the iterative heuristic method in the GA

5.5 RESULTS AND DISCUSSION

Data extracted from Short-term Hydro Optimization Program (SHOP) runs are used to compare the entire methods presented in the previous section. SHOP is an optimization model for planning hydropower systems provided as software by SINTEF Energy Research. SHOP is capable of addressing a wide range of operational, physical, and market constraints within hydro systems [19, 99]. Two power plants are connected in series in this case study. The first power plant has two turbines with a maximum reservoir volume of 41.66 Mm^3 and a maximum energy production capacity of 240 MWh. The second power plant consisting of four turbines, has a maximum energy output of 345 MWh, and has a maximum reservoir volume of 104.16 Mm^3. In this section, the parameters of the genetic algorithm are presented, followed by a comparison of three methods and their numerical results. Subsequently, the performance of the proposed methods is evaluated by comparing them to the optimal solution. All methods are implemented in Julia [123], and the optimization software to solve NLP is Ipopt [111]. In order to solve the models, a laptop computer equipped with an Intel Core i5 processor and 8 GB of RAM is used.

According to the data obtained from SHOP, which includes reservoir volume, water discharge, and power production, the maximum power output surfaces are obtained for both powerhouses. Instead of considering all turbine combinations, two output surfaces are used for the first powerhouse when one and two turbines are active, and four output surfaces for the second powerhouse when one, two, three, and four turbines are active. To obtain the nonlinear equations for the maximum power output surface for each number of active turbines, a polynomial approximation is fitted to the data. The planning horizon is 24 hours, and prices and inflows are deterministic.

5.5.1 PARAMETERS OF THE GENETIC ALGORITHM

In the presented binary genetic algorithm, approaches of one-point, two-point, and uniform form crossovers and mutations are employed to explore the problem space. At each stage, the previous population combines with the new generation population from crossover and mutation, and after evaluating the population, the next population in the genetic algorithm is created. Statistical methods have been used to calculate the parameters of the genetic algorithm in such a way that the problem is solved across various parameters. The parameters selected for the genetic algorithm are those that not only yield better results in terms of the average value of the objective function but also converge to the result in fewer iterations and solution time. Considering an investigation of the problem under various input conditions and constraints, as well as the number of iterations and solution runtime, a crossover rate of 0.8 and a mutation rate of 0.3 are adopted for the genetic algorithm.

5.5.2 NUMERICAL RESULTS

The methods were tested on 54 instances with different input parameters when reservoirs were almost empty, half full, and full. In order to evaluate the methods, the problem with different demand constraints was investigated and the demand graph for different hours can be seen in Figure 5.5.

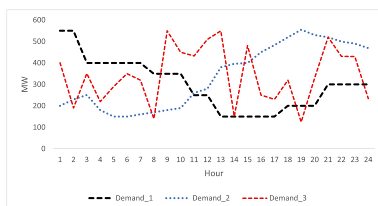

Figure 5.5 : Demand constraint for different hours

The results of instances are presented in Table 5.2, which includes the revenue, the number of iterations and computation time for each method. Each method was repeated five times for each instance, and the average value of the objective function, the average number of iterations, and the average execution time are reported.

Due to the use of maximum output surfaces rather than all turbine combinations, the number of problem states and the complexity of the problem are reduced, and the presented methods converge to the result after a suitable number of iterations. As mentioned earlier, Method *A* searches the problem space entirely randomly, while Method *B* determines the number of active turbines using rules after an initial guess. Method *C* uses both methods *A* and *B* to reach the result. Table 5.2 shows method *C* has a better value of the objective function in most instances than methods *A* and *B*. In cases 13, 16 and 38, method *A* has better results than

Table 5.2 : **Comparison of the results obtained using all three methods.**

Demand	Instance	Inflows	Reservoir 1	Reservoir 2	Method A			Method B			Method C		
					Obj(EUR)	Iterations	Time(s)	Obj(EUR)	Iterations	Time(s)	Obj(EUR)	Iterations	Time(s)
1	1	High	Half full	Half full	601,545	108	117.6	593,965	5	1.7	**602,310**	44	86.7
	2		Almost full	Almost full	806,073	82	122.7	805,695	6	1.6	**806,435**	29	66.3
	3		Almost empty	Almost full	525,588	71	165.8	515,057	6	1.9	**526,265**	44	90.0
	4		Almost full	Almost empty	552,653	70	113.7	551,018	6	1.7	**553,446**	28	43.4
	5		Half full	Almost full	433,352	79	201.5	427,139	6	1.9	**435,158**	37	85.7
	6		Almost full	Half full	539,930	75	136.8	527,273	5	1.5	**540,730**	45	80.8
	7	Medium	Half full	Half full	575,440	76	141.8	569,284	5	1.5	**576,630**	33	58.2
	8		Almost full	Almost full	780,077	79	137.6	779,181	6	1.8	**780,781**	25	43.9
	9		Almost empty	Almost full	496,614	68	167.8	489,572	5	1.6	**497,049**	42	98.4
	10		Almost full	Almost empty	530,739	58	100.4	529,697	6	1.8	**531,466**	30	53.6
	11		Half full	Almost full	401,425	69	174.3	394,923	6	1.9	**402,568**	37	92.4
	12		Almost full	Half full	513,449	68	119.6	502,238	7	2.1	**513,813**	68	121.8
	13	Low	Half full	Half full	548,741	79	137.5	542,439	5	1.5	**549,146**	30	54.5
	14		Almost full	Almost full	753,176	90	139.5	752,783	6	1.7	**753,573**	29	45.2
	15		Almost empty	Almost full	466,260	68	164.1	454,840	5	1.5	**467,652**	43	99.3
	16		Almost full	Almost empty	508,131	68	120.5	505,692	5	1.5	**508,229**	40	68.9
	17		Half full	Almost full	367,048	67	172.2	361,844	6	1.3	**368,559**	37	93.3
	18		Almost full	Half full	485,402	81	161.6	473,349	4	1.8	**485,88**	46	87.8
2	19	High	Half full	Half full	550,390	78	221.7	549,045	7	2.2	**551,923**	37	96.0
	20		Almost full	Almost full	**748,408**	73	103.0	747,262	5	1.5	748,388	25	35.8
	21		Almost empty	Almost full	476,642	60	183.6	470,783	5	1.8	**477,856**	35	95.7
	22		Almost full	Almost empty	496,276	75	200.0	495,382	4	1.3	**496,772**	28	75.5
	23		Half full	Almost full	383,410	77	155.1	374,026	6	2.2	**384,141**	39	105.4
	24		Almost full	Half full	491,362	71	175.4	484,971	6	2.0	**492,818**	44	107.9
	25	Medium	Half full	Half full	525,778	79	212.5	521,690	6	2.1	**526,979**	31	84.0
	26		Almost full	Almost full	722,661	70	101.1	722,426	6	1.8	**723,357**	33	51.2
	27		Almost empty	Almost full	448,815	68	196.2	441,392	6	2.2	**449,899**	43	95.2
	28		Almost full	Almost empty	474,881	70	198.3	474,422	5	1.7	**475,182**	24	55.9
	29		Half full	Almost full	347,837	83	207.3	340,270	5	1.8	**349,494**	35	84.0
	30		Almost full	Half full	**465,842**	90	207.6	455,564	6	2.1	465,369	42	96.0
	31	Low	Half full	Half full	499,988	79	225.7	497,338	4	1.5	**500,852**	31	81.2
	32		Almost full	Almost full	696,455	78	116.4	695,588	6	1.8	**697,015**	28	40.4
	33		Almost empty	Almost full	419,410	74	200.8	410,483	5	1.9	**419,624**	45	120.6
	34		Almost full	Almost empty	451,774	78	195.8	452,342	5	1.8	**453,321**	35	80.4
	35		Half full	Almost full	**311,959**	92	238.1	303,914	6	2.2	311,941	39	99.6
	36		Almost full	Half full	438,173	86	301.8	428,579	5	1.8	**438,620**	42	108.2
3	37	High	Half full	Half full	**553,271**	76	198.3	547,866	6	2.0	553,143	35	96.8
	38		Almost full	Almost full	754,713	68	92.0	754,554	5	1.7	**754,927**	28	42.2
	39		Almost empty	Almost full	474,909	72	203.7	464,428	5	2.0	**474,965**	59	169.9
	40		Almost full	Almost empty	500,866	83	211.3	500,479	6	2.4	**501,532**	38	104.2
	41		Half full	Almost full	372,600	76	182.6	367,112	6	2.3	**372,773**	51	134.5
	42		Almost full	Half full	490,679	80	194.8	476,443	6	2.3	**491,222**	56	137.4
	43	Medium	Half full	Half full	526,321	77	198.6	521,765	6	2.4	**526,630**	39	94.1
	44		Almost full	Almost full	729,370	79	96.8	728,377	5	1.8	**729,764**	29	37.8
	45		Almost empty	Almost full	443,992	73	198.1	431,699	5	2.0	**444,251**	48	128.7
	46		Almost full	Almost empty	478,745	71	180.5	477,731	5	2.0	**478,86**	28	70.7
	47		Half full	Almost full	335,362	82	207.1	329,323	5	2.1	**335,421**	44	93.7
	48		Almost full	Half full	462,627	68	186.9	446,114	6	2.3	**463,026**	67	172.6
	49	Low	Half full	Half full	498,660	72	201.7	493,868	6	2.5	**499,336**	55	154.4
	50		Almost full	Almost full	702,566	74	100.1	701,969	6	2.1	**703,347**	26	39.6
	51		Almost empty	Almost full	411,372	75	194.8	404,818	5	2.1	**411,764**	58	176.0
	52		Almost full	Almost empty	455,593	74	178.3	453,572	6	2.6	**455,827**	40	105.6
	53		Half full	Almost full	296,378	67	161.2	291,358	6	2.5	**297,179**	45	103.2
	54		Almost full	Half full	**432,999**	81	225.0	417,161	6	2.5	432,802	49	125.8

method C, although their difference is also quite small. The objective function value in method C is on average 0.09% greater than method A and 0.63% greater than method B. According to a comparison of the solution times, method B has the shortest execution time and converges to the result in the shortest number of iterations, while method C has a shorter execution time and fewer iterations than method A. According to the results, the average calculation time for methods A, B, and C is 138, 2 and 74 seconds, respectively. and the average number of iterations is 78, 5, and 42, respectively. The average number of iterations in method A is 78, method B is 5, and method C is 42.

The objective function for three methods are compared using Eq.(5.16), in which the result of each method is divided by the maximum value obtained in each method. For example, the maximum value in instance 1 in Table 5.2 is 447,910 (EUR), which is obtained from method C. Therefore, the results in instance one are divided by the maximum value. Figure 5.6 shows the percentage of similarity of each method with the best result. Accordingly, any method with the best result will have the same numerator and denominator in equation Eq.(5.16), and the percentage of similarity will be 100 %. As illustrated in the Figure 5.6, in most instances, method C obtained the best results. Despite method B has weaker results than the other two methods, and it has a maximum difference of 1.60% in instance 53.

$$\text{Similarity with the best result}(\%) = \left(\frac{\text{The objective function of the method A, B and C (EUR)}}{\text{Maximum objective function (EUR)}}\right) \times 100 \quad (5.16)$$

In order to compare the number of iterations of methods B and C the instances 5, 11, 31, and 47 were randomly selected from table 5.2 and results are shown in Figure 5.7. In Figure 5.7, the horizontal axis represents the number of iterations and the vertical axis represents the value of the objective function. The result of method C converges with a smaller number of iterations compared to method B.

In Figure 5.8, the volume of the first and second reservoirs, along with the water discharge of plants 1 and 2, across three methods in instance 5, is compared. Based on Eq.(5.6)

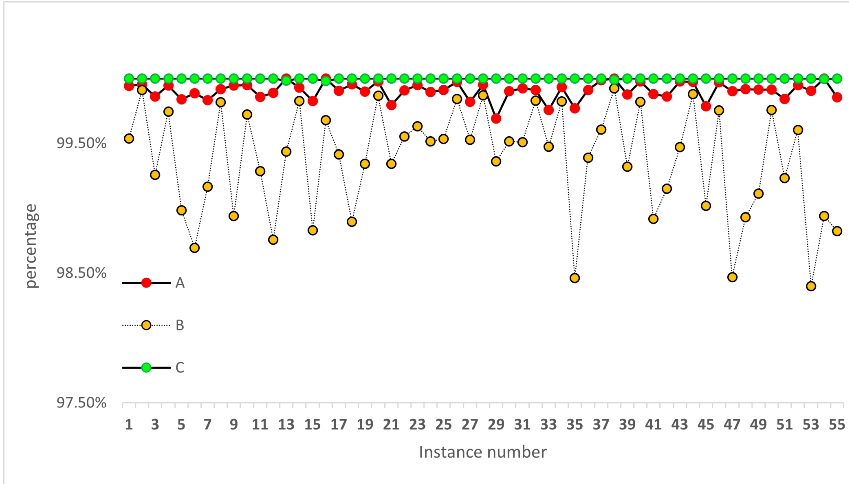

Figure 5.6 : Comparison of the value of the objective function in three methods

in the model presented, and considering the penalty for insufficient demand coverage, all three methods effectively meet the demand constraints. According to the price parameter in the model and after fulfilling the demand constraint, power production is conducted at higher prices, as illustrated in Figure 5.9. Furthermore, it has been demonstrated that Method *B* incurs higher start-up costs compared to methods *A* and *C*, as the approach for determining the number of active turbines in Method *B* focuses on maximizing production.

The results show that method *C* has a shorter execution time than method *B* and that the objective function value is higher in most cases. For large-scale problems and situations where exact solutions are not feasible, method *B* is a viable option to obtain results expeditiously. By using a heuristic approach in genetic algorithms, method *C* utilizes the advantages of methods *A* and *B*, and brings good chromosomes into the problem solving process. As a result, method *C* can improve their performance and allow them to achieve better results in a shorter period of time.

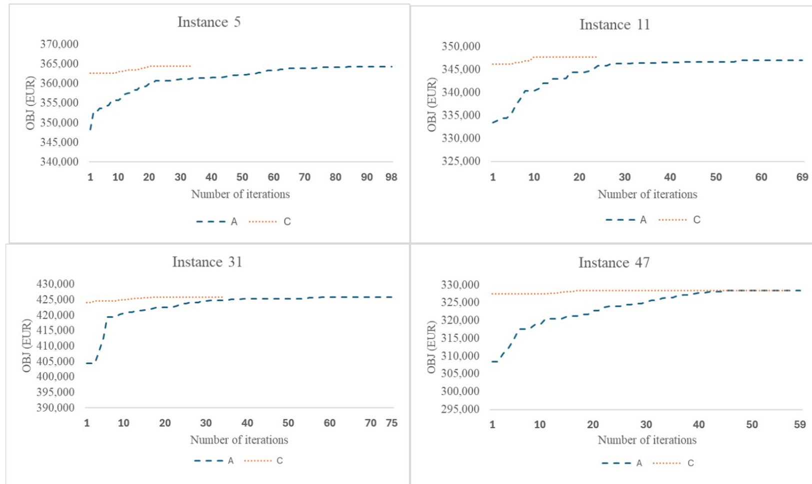

Figure 5.7 : Comparison of the changes in the objective function value in each iteration, instances 5, 11, 31 and 47.

Despite the fact that basic operators were employed in the genetic algorithm, the results indicate that this approach can exhibit considerable efficiency despite its simplicity. A more effective method and technique for exploring a problem space can undoubtedly lead to improved outcomes and solution times.

5.5.3 BENCHMARK VALIDATION

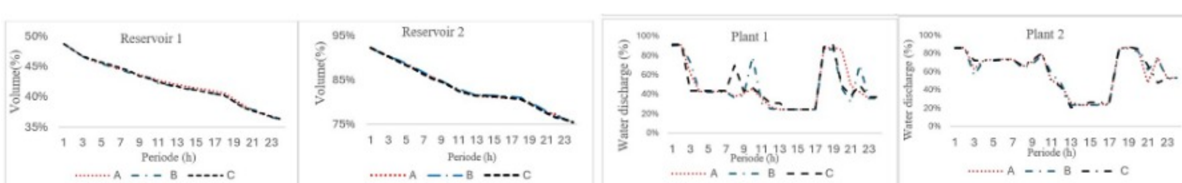

Figure 5.8 : Comparison of reservoir volume and Water discharge across three methods: Instance 5

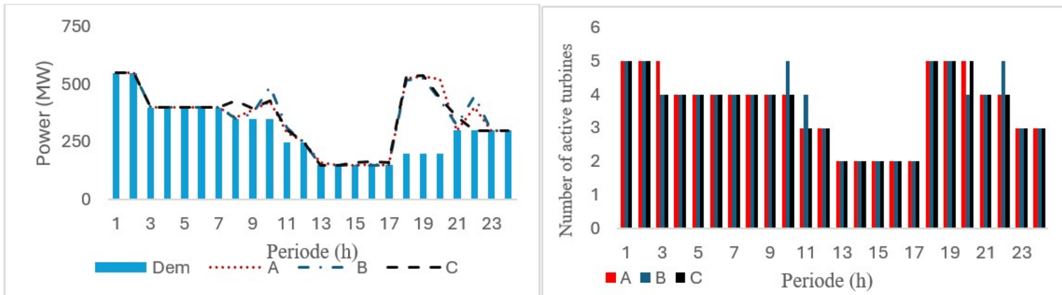

Figure 5.9 : Comparison of power production in three methods with Demand coverage and number of active turbines: Instance 5

The MINLP short-term hydropower problem can be solved if the totally unimodular condition is satisfied, as illustrated in [8]. Therefore, to compare methods A, B, and C with the optimal solution, some model conditions, including start-up costs, and demand constraints, are ignored. Thus, the loading problem formulation is taken from [8] to evaluate the methods and is as follows:

$$\max \sum_{c\in C}\sum_{t\in T}\sum_{j\in J} \rho_t \times \chi^c_{j,t}(q^c_t, v^c_t) \times z^c_{j,t} \tag{5.17}$$

Subject to:

$$v^c_{t+1} = v^c_t - \zeta \times w_t \times (q^c_t + g^c_t) + \zeta \times \delta_t + \sum_{r\in R} \zeta \times w_t \times (q^r_t + g^r_t) \quad , \quad \forall t \in T, c \in C, \tag{5.18}$$

$$\sum_{j\in J} z^c_{j,t} = 1 \quad , \quad \forall t \in T,\, c \in C, \tag{5.19}$$

$$q^c_{min} \leq q^c_t \leq q^c_{max} \quad , \quad \forall t \in T,\, c \in C, \tag{5.20}$$

$$v^c_{min} \leq v^c_t \leq v^c_{max} \quad , \quad \forall t \in T,\, c \in C, \tag{5.21}$$

$$v^c_1 = v^c_{Initial} \quad , \quad \forall\, c \in C, \tag{5.22}$$

$$v^c_T = v^c_{final} \quad , \quad \forall\, c \in C, \tag{5.23}$$

$$v^c_t \geq 0,\; q^c_t \geq 0 \quad , \quad \forall t \in T,\, c \in C, \tag{5.24}$$

$$z^c_{j,t} \in B \quad , \quad \forall t \in T,\, j \in J,\, c \in C. \tag{5.25}$$

Five instances from Table 5.2 were randomly selected for comparison, and Eq.(5.26) was used to determine the percentage of similarity with the optimal value.

$$\text{Similarity}(\%) = \left(\frac{\text{Avarage of objective function of presented method (EUR)}}{\text{Objective function of Loading problem (EUR)}}\right) \times 100 \tag{5.26}$$

It was repeated five times in each method in order to obtain the average value of the objective function for each instance. The percentage of similarity for each instance is shown in Table 5.3.

Table 5.3 : Comparison of all three methods with the optimal solution.

Instance	Inflows	Reservoir 1	Reservoir 2	Method A (%)	Method B (%)	Method C (%)
10	Medium	Almost full	Almost empty	99.99%	99.95%	99.99%
21	High	Almost empty	Almost full	99.97%	99.49%	99.98%
29	Medium	Half full	Almost full	99.98%	99.40%	99.99%
42	High	Almost full	Half full	99.97%	99.25%	99.98%
49	Low	Half full	Half full	99.99%	99.72%	99.98%

The results show that Method *A*, Genetic Algorithms, and Method *C*, which utilizes a heuristic algorithm within Genetic Algorithms, are capable of approaching the optimal solution very well, and the difference between them and the optimal solution is relatively small. Although it was demonstrated that method *B* has reasonable accuracy to reach the result and can reach the solution in a short period of time in different conditions of the input parameters, the initial guess can affect the results and this method does not perform well at breaking points, as mentioned previously.

5.6 CONCLUSION

In this paper, a MINLP for short-term hydropower problems with demand constraints and start-up costs is presented. The complexity of the problem is reduced significantly by fixing the binary variable, the number of active turbines, and using the maximum power output surface. Thus, rather than estimating other variables, the exact solver was used to solve the nonlinear problem, and three methods were presented. In method *A*, a binary genetic algorithm was used to solve the nonlinear problem. In method *B*, an iterative heuristic approach was employed to determine the number of active turbines and solve the problem. An iterative heuristic approach was applied to the genetic algorithm in method *C*. Based on the results, method *C* utilizes the advantages of both methods, searches the problem space well, and converges to the result with fewer iterations than method *A*. The average result from method *C* is 0.09% better than method *A* and 0.63% better than method *B*. For future studies, uncertainties such as prices and inflows can be taken into account in the model and solution methods. Metaheuristic algorithms such as PSO, AC, and SA, can also be used to solve this problem and it would be interesting to compare the results with methods developed in this paper.

CHAPTER VI

OPTIMIZING PROFILE BLOCK BIDS IN SHORT-TERM HYDROPOWER SCHEDULING: A TWO-PHASE MODEL FOR THE DAY-AHEAD MARKET

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6.1 ABSTRACT

This paper proposes a two-phase optimization framework for short-term hydropower scheduling in the day-ahead electricity market using profile block bids grouped in exclusive sets. The first phase solves a nonlinear deterministic model that generates a diverse and operationally feasible set of production blocks by accounting for start-up costs, opportunity costs, and hydrological constraints. In the second phase, a two-stage stochastic program is used to select a subset of blocks for market submission under price uncertainty. The proposed approach captures a wide range of production scenarios while ensuring compliance with market design rules. By decomposing the problem and relaxing binary variables, the framework significantly reduces computational complexity and achieves fast solution times. Numerical experiments based on a real hydropower system demonstrate the model's ability to produce effective bidding strategies, comparable to the hourly bidding methods.

NOTATION

Sets

$t \in \{1,2,\ldots,T\}$	Index of periods
$c \in \{1,2,3,\ldots,C\}$	Index of hydropower plants
$b \in \{1,2,3,\ldots,B_c\}$	Index of block for plant c
$s \in \{1,2,\ldots,S\}$	Index of scenarios
$r \in \{1,2,\ldots,R^c\}$	Index of plants upstream of plant c
$j \in \{1,2,\ldots,J_t^c\}$	Index of power production surface j for plant c at period t

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Parameters

ζ	Conversion factor from (m^3/s) to (Mm^3/h)
q^c_{min}	Minimum water discharge at plant c (m^3/s)
q^c_{max}	Maximum water discharge at plant c (m^3/s)
v^c_{min}	Minimum volume of plant c reservoir (Mm^3)
v^c_{max}	Maximum volume of plant c reservoir (Mm^3)
$v^c_{Initial}$	Initial volume of reservoir c (Mm^3)
$P_{t,b}$	Price of profile b at hour t (Phase 1)
ε^c	start-up penalty for plant c
δ^c_t	Inflow in period t (Mm^3)
$\rho_{s,t}$	Market price for scenario s at hour t
γ	Penalty for each additional selected profile beyond the first one
π^s	Probability of scenario s
$\tau^{r\rightarrow c}$	Time delay (h) for water transfer from upstream reservoir r
R_{bs}	Revenue from using profile b in scenario s
C_s	Cost of not selecting any profile in scenario s

Variables

q_t^c	Water discharge at period t (m^3/s)
v_t^c	Reservoir volume at period t (Mm^3/h)
g_t^c	Water spillage at plant c and period t (m^3/s)
$\alpha_t^c(v_t^c)$	Opportunity cost of water usage at plant c during period t
$\chi_{j,t}^c$	Power production for surface j (MW)
oc^c	Lost opportunity cost of using water from plant c at period t
z_j	$\begin{cases} 1, & \text{if surface } j \text{ is chosen} \\ 0, & \text{otherwise} \end{cases}$
x_{bs}	$\begin{cases} 1, & \text{if profile } b \text{ is in the exclusive group in scenario } s \\ 0, & \text{otherwise} \end{cases}$
w_b	$\begin{cases} 1, & \text{if profile } b \text{ is selected} \\ 0, & \text{otherwise} \end{cases}$
u_s	$\begin{cases} 1, & \text{if no profile is selected in scenario } s \\ 0, & \text{otherwise} \end{cases}$

6.2 INTRODUCTION

Bids can be structured in various formats in the electricity market, such as hourly bids, flexible hourly bids, and block bids that group multiple hours together. Block bids are used in systems with intertemporal dependencies between reservoirs, where water flow delays can cause mismatches between production and market prices. Block bids contribute to stable production over longer time periods and are particularly suitable in situations where there are conflicts between upstream and downstream hydropower plants [13]. Block bidding provides producers in electricity markets with an organized method to manage operational restrictions and cost considerations effectively. Block bidding offers a structured approach to managing the complexities of power markets by taking into account operational limitations and cost considerations. These bids facilitate conditional and time-linked power delivery, ensuring better alignment between production schedules and market demand. Block bids increase stability and reduce inefficiencies, especially under complex conditions[124].

Block bids, characterized by an all-or-nothing acceptance condition, enable conditional and intertemporal power delivery. These bids address market and operational needs through various types. Regular block bids deliver constant power over a specific period, profile block bids enable variable energy profiles, and linked block bids establish parent-child relationships for conditional acceptance [124, 125, 126]. Profile block orders, unlike regular block orders, allow producers to offer variable energy quantities across multiple periods, aligning delivery with market price fluctuations. Their clearing relies on comparing the offered price with the weighted average market clearing price over the selected intervals [127].

Several studies have explored the integration of block bids into the offering strategies of hydropower producers. For instance, [92] investigates the use of regular block orders in hourly offering problems, focusing on their role in addressing operational constraints like start-up and shutdown costs. Similarly, [97] examine day-ahead bidding strategies, incorporating both hourly and block bids to optimize production while managing price uncertainties and balancing market dynamics. Alnæs et al. [128] further provide an empirical analysis of Norwegian hydropower producers, highlighting the interplay between block and hourly bids and their effectiveness in addressing marginal water values and operational efficiencies. In [129], a day-ahead planning model integrates stochastic programming and recurrent neural networks, addressing hourly and block bids with price uncertainties for hydropower producers in the Nordic market. In [130], block bids are optimized for combined heat and power units using stochastic programming, highlighting their role in managing price uncertainties and operational constraints. Karasavvidis et al. [124] extend previous research by developing an optimization framework for hydrothermal systems that incorporates advanced bidding structures, such as profile-based and linked block bids. These approaches address operational challenges, improve production flexibility, and optimize profits under varying price and regulatory conditions. Recent developments in electricity market design have highlighted the role of exclusive groups of block bids, also known as mutually exclusive block bids, in addressing intertemporal constraints and enabling more realistic representations of production capabilities. These bid formats allow participants to submit multiple alternative power profiles while ensuring that at most one of them is accepted, thus preventing overcommitment and supporting better operational alignment. In this context, [9] present an in-depth analysis of package bidding mechanisms used in European electricity auctions. Their work is particularly relevant to our study, as the two-phase modeling approach we adopt, especially the second-phase optimization problem characterized by total unimodularity, is inspired by their formulation. They show how the limitation on the number of allowable block bids can lead to welfare losses and propose algorithmic solutions to optimize bid selection under these constraints. Their insights form a theoretical foundation for our stochastic profile selection model.

As highlighted in the literature, most existing studies on block bidding focus on regular block bids, while fewer works address the profiled block bids. In this paper, we present a two-phase model for profiled block bidding for participating in the Norwegian day-ahead market. In the first phase, a short-term hydropower optimization problem is solved, incorporating operational constraints, water usage costs, and start-up costs. Since the model can handle various price conditions and sequences of consecutive hours, it is capable of generating diverse and realistic bidding blocks—an important feature for dealing with complex systems. The second phase is a two-stage linear stochastic program that selects the most suitable blocks to offer to the market from the block set generated in the first phase, based on the price scenarios. The two-phase structure of the approach causes a reduction in computational time, and the model converges to the solution within a short period.

This paper is organized as follows: Section 6.3 presents the methodology and mathematical formulation of the two-phase model, including the profile generation model developed in Phase 1 and the two-stage stochastic profile selection model of Phase 2. Section 6.4 introduces a case study and the hydropower system. The results are reported in Section 6.5, and the performance of the model is compared to the hourly bidding strategies to evaluate its effectiveness in Section 6.6. Finally, Section 6.7 presents the conclusion and directions for future research.

6.3 METHODOLOGY

Short-term hydropower scheduling involves determining the optimal hourly production plan for one or more hydro units over a short planning horizon, typically ranging from one to several days. This planning must consider operational constraints such as reservoir storage bounds, turbine operating limits, and start-up costs. Power production depends on technical factors including reservoir volume, discharge, and net head. The main objective is to allocate water resources in a way that ensures feasible operation while responding effectively to market price variations [8, 17]. Electricity trading is commonly organized into three main markets: the day-ahead market, the intraday market, and the balancing (real-time) market. The day-ahead market main role where most electricity transactions are conducted. In this market, producers and consumers submit their bids for the following day, typically before noon. After collecting all offers, the market operator performs market clearing and publishes the hourly market prices and committed quantities around 1 p.m. [13, 56]. The intraday market allows participants to update their positions closer to delivery, offering increased flexibility in response to new forecasts or unexpected changes. Finally, the balancing market, operated by the Transmission System Operator (TSO), is used to resolve real-time imbalances and ensure system stability. In this market, producers can offer flexible ramping capacity or make adjustments based on the actual system conditions [13]. Producers can submit different types of bids to participate in these markets. In addition to hourly bids, market designs increasingly allow for more structures such as block bids, which consist of fixed quantities over multiple hours with an all-or-nothing acceptance rule. These include regular block bids (constant quantity over time), profile block bids (varying quantity), and linked block bids that define conditional relationships between bids. Some market designs allow producers to submit several alternative block bids as part of an exclusive group, with the rule that only one of them can be accepted. This structure prevents overcommitment and allows producers to adapt their bidding strategy to different possible operating conditions [9].

This paper proposes a two-phase optimization framework to identify an optimal profile block bidding strategy for short-term hydropower scheduling in the day-ahead electricity market under price uncertainty. The methodology is designed to ensure operational feasibility while maximizing market-based profitability. In the first phase, a deterministic optimization model is solved using forecasted electricity prices and inflow data. This model incorporates key operational features such as opportunity costs, start-up costs, and hydrological constraints including reservoir balance and turbine limits. The goal is to generate a diverse and feasible set of production profiles (blocks) that are compliant with the rules of block bidding in electricity markets. Each block represents a continuous production period with durations ranging from a minimum of 3 consecutive hours up to 24 hours. This diversity allows the model to accommodate various operational conditions and prepares it to respond flexibly to a wide range of market scenarios.

The feasible blocks generated in this phase, along with their associated production costs, opportunity costs, and start-up costs, are passed to the second phase. In this phase, a two-stage stochastic programming model is used to select the most profitable subset of blocks for market participation. Price uncertainty is modeled through a set of price scenarios that become available close to the bidding deadline. Based on these scenarios, the model evaluates the expected economic performance of each block and selects a fixed number (e.g., 15 blocks) that maximize the overall expected profit. This selection process reflects actual market design, where producers are typically allowed to submit a limited number of block bids grouped into exclusive sets. In such exclusive groups, only one block can be accepted per scenario. The objective accounts for all relevant costs, including production, opportunity, and start-up costs. The proposed formulation ensures computational efficiency, even for large-scale instances, and provides a structured and scenario-driven approach to support informed bidding decisions under uncertainty.

6.3.1 PHASE 1: PROFILE GENERATION

Phase 1 of the proposed methodology focuses on generating a set of profile block bids that define potential operational schedules for the hydropower plant over a given time horizon. To achieve this, a nonlinear deterministic optimization model is formulated, in which market prices and inflows are considered as parameters. The objective is to maximize revenue while accounting for key operational costs, including water usage, opportunity costs, and turbine start-up expenses. The model ensures efficient water resource allocation while respecting hydrological and operational constraints.

Hydropower optimization is inherently nonlinear, as power production depends on water discharge, reservoir volume, and turbine efficiency. The net water head, which directly influences power generation, is determined by the forebay and tailrace elevations, as well as penstock losses. Additionally, turbine efficiency varies across units, meaning that even under similar water discharge and head conditions, different turbines may yield different power outputs. Instead of explicitly modeling each turbine configuration, the model employs the maximum power output surface, which approximates the nonlinear relationship between water discharge and reservoir volume and power production using polynomial regression. These power output surfaces, derived from a combination of feasible turbine operations, provide a computationally efficient way to capture the complexities of turbine efficiency and head variations. Instead of modeling each turbine individually and considering all possible configurations, the model utilizes the maximum power output surface, which simplifies the representation of power generation while maintaining accuracy.

The inclusion of power output surfaces introduces binary variables, leading to a Mixed-Integer Nonlinear Programming (MINLP) formulation. While MINLP models provide precise solutions, they can be computationally expensive, particularly in large-scale hydropower systems. To improve tractability, the model is formulated in a way that ensures total unimodularity in the constraint matrix. As a result, even when binary variables are relaxed, the problem still yields integer solutions.

In this case, the matrix of the coefficients of the constraints is totally unimodular and therefore meets these three criterias to be defined so: 1) All submatrices have elements in the set {-1, 0, 1}. 2) Each column has at most two nonzero elements. 3) There exists a partition of rows such that every column with two nonzero elements satisfies this partition. If these conditions are met, the binary selection problem can be solved as a continuous nonlinear problem while still yielding integer solutions. The optimization model aims to maximize total revenue by selecting the most efficient production profiles while accounting for key operational costs. The objective function Eq (6.1) maximizes the total profit, where P_t denotes the market price at time t, and $\chi^c_{j,t}(q^c_t, v^c_t)$ represents the power output as a function of water discharge and reservoir volume. An essential component of the model is the inclusion of water usage costs or opportunity costs. These costs are modeled using a linear function that depends on both reservoir volume and water discharge. The intuition behind this is straightforward: when the reservoir is near full capacity, the opportunity cost of using water is low, as there is little risk of scarcity. However, as the water level drops, the opportunity cost increases, reflecting the growing value of conserving water for future use. This dynamic encourages more strategic water allocation, especially during periods of low storage. In addition, start-up costs are included for each production block. These costs are determined by solving a unit commitment problem, following the methodology described in [8]. The mathematical formulation of Phase 1 is presented as follows.

$$\max \sum_{c\in C}\sum_{t\in T}\sum_{j\in J} P_t \times \chi^c_{j,t}(q^c_t, v^c_t) \times z^c_{j,t} - \sum_{c\in C}\sum_{t\in T} \alpha^c_t(q^c_t, v^c_t) \tag{6.1}$$

Subject to:

$$\begin{aligned}
v^c_{t+1} = \quad & v^c_t - \zeta \times w_t \times (q^c_t + g^c_t) + \zeta \times \delta^c_t \\
& + \textstyle\sum_{r\in R} \zeta \times w_{t-\tau^{r\to c}} \times (q^r_{t-\tau^{r\to c}} + g^r_{t-\tau^{r\to c}}) \\
& , \forall t \in T, c \in C, && (6.2)\\
\sum_{j\in J} z^c_{j,t} \leq 1 \quad & , \forall t \in T, c \in C, && (6.3)\\
v^c_1 = v^c_{Initial} \quad & , \forall c \in C, && (6.4)\\
q^c_{min} \leq q^c_t \leq q^c_{max} \quad & , \forall t \in T, c \in C, && (6.5)\\
v^c_{min} \leq v^c_t \leq v^c_{max} \quad & , \forall t \in T, c \in C, && (6.6)\\
v^c_t \geq 0, q^c_t \geq 0 \quad & , \forall t \in T, c \in C, && (6.7)\\
z^c_{j,t} \in \{0,1\} \quad & , \forall t \in T, j \in J, c \in C. && (6.8)
\end{aligned}$$

Eq. (6.2) defines the water balance for each reservoir in the system. It ensures that the volume of water stored in reservoir c at time $t+1$, denoted by v^c_{t+1}, is equal to the volume at time t, v^c_t, minus the water released for power production and spillage, $w_t(q^c_t + g^c_t)$, plus the natural inflow δ^c_t, all scaled by the conversion factor ζ, which converts discharge from m^3/s to Mm^3/h. Additionally, the equation accounts for water inflow from upstream reservoirs that are hydraulically connected to reservoir c. These contributions are modeled with a delay $\tau^{r \to c}$, representing the travel time of water from an upstream reservoir r to reservoir c. This formulation provides a realistic representation of reservoir interactions, especially in systems where water released from upstream plants does not immediately reach downstream reservoirs. Equation (6.3) guarantees that, for each unit c and every time period t, exactly one production surface is selected from the available set. Equation 6.4 sets the initial reservoir volume to a predefined value $v_1^c = v_{Initial}^c$, ensuring a known starting condition. Equations 6.5 and 6.6 impose operational constraints on water discharge and reservoir volume, restricting them within their respective minimum and maximum limits to maintain system feasibility. Equation 6.7 enforces non-negativity constraints on reservoir volume and water discharge to ensure physically meaningful solutions. Finally, Equation 6.8 defines the binary nature of $z_{j,t}^c$.

6.3.2 PHASE 2: TWO STAGE STOCHASTIC PROFILE SELECTION OPTIMIZATION

The objective of Phase 2, a two-stage stochastic mixed-integer linear programming model, is to select the optimal blocks based on new price scenarios from among the block set generated in Phase 1. Thus, the optimal power production values and associated block costs calculated in Phase 1 are considered as inputs for Phase 2. Additionally, the scenarios incorporate the uncertainties of day-ahead market prices. The objective function in Phase 2 includes revenue from each block under different price scenarios, deducts associated costs—such as opportunity and start-up costs, and incorporates a penalty term to prevent the selection of blocks that do not contribute to improving the objective function value. The mathematical formulation of the second phase is as follows:

$$\max \sum_{b \in B} \sum_{s \in S} \pi^s R_{bs} x_{bs} - \sum_{s \in S} \pi^s C_s u_s - \gamma \left(\sum_{b \in B} w_b - 1 \right) \tag{6.9}$$

$$\sum_{b \in B} w_b \leq N_{blocks}, \tag{6.10}$$

$$\sum_{b \in B} x_{bs} + u_s = 1 \qquad , \forall s \in S, \tag{6.11}$$

$$x_{bs} \leq w_b \quad , \forall b \in B, \forall s \in S, \tag{6.12}$$

$$w_b \in \{0,1\} \qquad , \forall b \in B, \tag{6.13}$$

$$x_{bs} \in \{0,1\} \quad , \forall b \in B, \forall s \in S. \tag{6.14}$$

$$\tag{6.15}$$

Equation (6.10) ensures that the total number of selected profiles does not exceed the predefined limit N_{blocks}, controlling the maximum number of bids submitted to the market. Equation (6.11) enforces that for each scenario, exactly one decision is made—either one of the available profiles is selected, or no profile is chosen, which is indicated by u_s. This guarantees that the sum of selections per scenario equals one. Equation (6.12) ensures that a profile b can only be selected in scenario s if it has already been included in the bidding set. This maintains logical consistency between the profile selection variable w_b and its scenario-dependent selection x_{bs}. Equation (6.13) specifies that each profile is either included in the bidding set or not, ensuring that no partial profile selections occur. Equation (6.14) enforces a binary decision on whether a profile is selected in a specific scenario, maintaining the discrete nature of the problem.

6.4 CASE STUDY

The two-phase model has been evaluated in a case study of a hydropower system in Norway, which includes multiple reservoirs and power plants. This system consists of six interconnected reservoirs (Sverjesjoen, Falningsjoen, Innerdalsvannet, Storfossdammen, Granasjoen, and Bjorsetdammen) that supply water to five hydroelectric power plants: Ulset, Litjfossen, Brattset, Grana, and Svorkmo. Each plant has specific generation capacities and water discharge constraints. The installed capacity varies across plants, with Brattset (88 MW), Grana (82.5 MW), and Litjfossen (84 MW) having the highest output potential, while Ulset (40 MW) and Svorkmo (57.7 MW) provide additional flexibility. The reservoirs also differ significantly in volume: Innerdalsvannet (153.4 Mm^3), Falningsjoen (125.2 Mm^3), and Granasjoen (138.8 Mm^3) offer substantial storage capacity, whereas smaller reservoirs such as Bjorsetdammen (0.02 Mm^3) and Storfossdammen (1.69 Mm^3) are primarily used for short-term regulation and discharge routing. The system's topology incorporates a network of bypass channels and spillways, which regulate water flow between reservoirs, enhancing operational flexibility and stability.

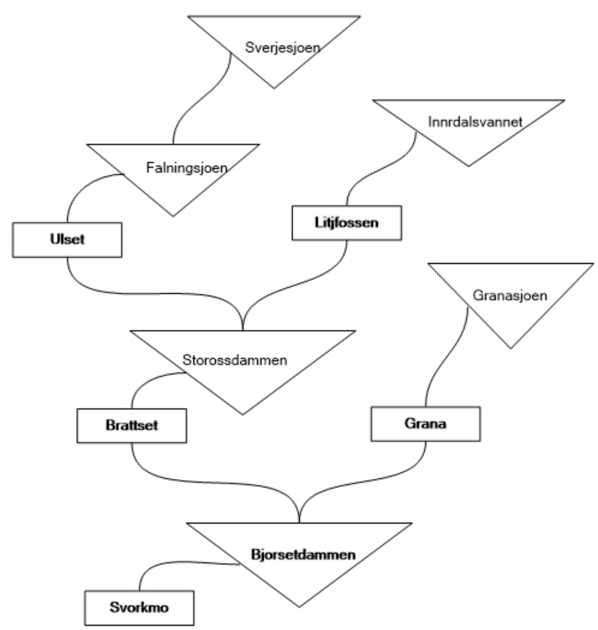

Figure 6.1 : System topology

The two-phase model involves solving two optimization problems with different structures. Phase 1 solves a short-term hydropower scheduling problem formulated as a mixed-integer nonlinear program (MINLP), aiming to generate feasible production profiles that account for water use, opportunity costs, and start-up costs. This problem is solved using the Ipopt solver [111].

Phase 2 uses a two-stage stochastic linear program to select the most profitable subset of blocks based on multiple day-ahead price scenarios. This model is solved with the CLP solver [131]. To validate the Phase 1 results, the BONMIN solver [109], which handles nonlinear problems with binary variables, is also used. The entire implementation is done in Julia [123], and the experiments are conducted on a system with an Intel Core i5 processor and 8 GB of RAM.

6.5 RESULTS

Since Phase 1 of the problem is solved deterministically, this section investigates the impact of the number of candidate blocks generated in Phase 1 on the performance of Phase 2. Specifically, we analyze how increasing the number of blocks in Phase 1 affects the objective value and the quality of the solution obtained in Phase 2.

To evaluate this, 5 representative days were randomly selected from the dataset. For each case, the stochastic second phase was solved using 30 day-ahead price scenarios. The corresponding results are summarized in Table 6.1, which reports both the objective function values and the computation times for different numbers of blocks.

Phase 1 was executed for various numbers of blocks: 25, 50, 100, 250, 500, 750, 1000, and 1500. The input parameters for Phase 1 include electricity prices and initial reservoir volumes. Each block is subject to predefined feasibility criteria, such as a minimum duration of 3 consecutive hours and a maximum of 24 hours. Any block that does not satisfy these criteria is excluded from the candidate set. The feasible blocks, along with their associated opportunity and start-up costs, are then passed to Phase 2. In this stage, a two-stage stochastic programming model is used to select the most profitable combination of blocks to be offered in the day-ahead electricity market. The price scenarios reflect real market conditions close to the bidding time.

As shown in Table 6.1, increasing the number of candidate blocks in Phase 1 generally leads to better objective function values in Phase 2. This indicates that having access to a richer set of block options improves bidding decisions under uncertainty.

Table 6.1 : Objective function value and computation time (in seconds) for different numbers of candidate blocks

		Number of blocks							
Case	**Measure**	**25**	**50**	**100**	**250**	**500**	**750**	**1000**	**1500**
Case 1	Time (s)	0.01	0.01	0.01	0.04	0.06	0.12	0.14	0.22
	Obj. Value	33479	34082	34608	36475	38521	38529	38540	38548
Case 2	Time (s)	0.01	0.00	0.01	0.02	0.03	0.05	0.14	0.15
	Obj. Value	59205.1	60458.5	61632.7	63676.3	65304	65843.9	65871.3	65871.3
Case 3	Time (s)	0.00	0.01	0.01	0.02	0.06	0.11	0.14	0.22
	Obj. Value	95381.2	97592	101592	103629	103629	104701	104701	104720
Case 4	Time (s)	0.01	0.01	0.01	0.02	0.04	0.09	0.10	0.11
	Obj. Value	13237.7	13237.7	13991.4	14308.8	15003.7	15043.7	15049.3	15079.7
Case 5	Time (s)	0.01	0.01	0.02	0.02	0.02	0.04	0.05	0.09
	Obj. Value	18178.3	18329.1	18784	19525.3	20468	20597.8	20617.7	20629

Figure 6.2 visualizes the normalized performance of Phase 2 based on the objective values presented in Table 6.1. The x-axis represents the number of candidate blocks used in Phase 1, while the y-axis shows the corresponding objective value expressed as a percentage of the maximum value (i.e., the value obtained with 1500 blocks, set to 100%).

These percentages were calculated by dividing the objective function value for each block size by the maximum value observed across all tested sizes, and then multiplying by 100. Formally, for a given number of blocks n, the normalized value is computed as:

$$\text{Percentage}_n = \left(\frac{Obj_n}{Obj_{\max}} \right) \times 100 \tag{6.16}$$

where Obj_n is the objective function value for n blocks, and $Obj_{\max}$ is the maximum value obtained (in this case, with 1500 blocks). As shown in the figure 6.2, increasing the number of candidate blocks leads to improvements in the objective function value. However, beyond 500 blocks, the improvements occur at a slower rate, indicating that the marginal benefit of adding more blocks diminishes.

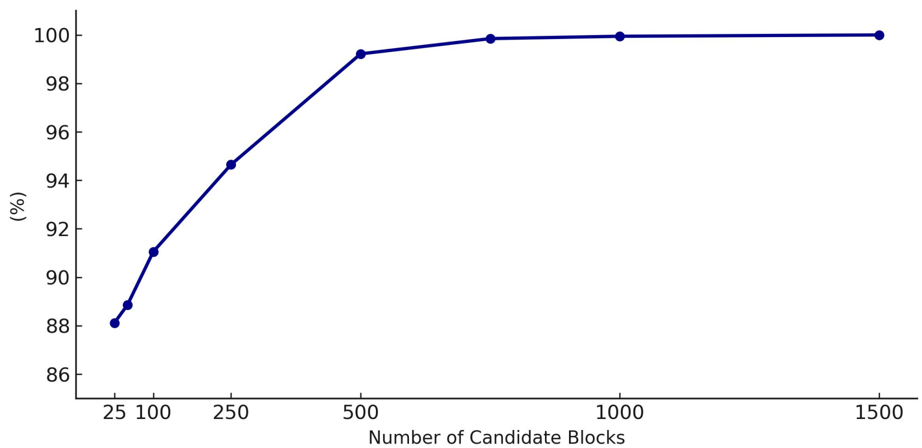

Figure 6.2 : Normalized objective function value for different numbers of candidate blocks.

Although the solution time increases slightly with the number of candidate blocks, Phase 2 remains computationally efficient. Even with up to 1500 blocks and 30 price scenarios, the problem can be solved in less than a second, which demonstrates the scalability of the proposed method.

Figure 6.3 illustrates the average solution time across five representative cases for different numbers of candidate blocks. As shown in the figure, the computational time grows gradually as more candidate blocks are introduced, but remains consistently low—well below one second—even for the largest problem sizes considered. This further confirms that the relaxation of binary variables, supported by the total unimodularity of the constraint matrices, enables fast and scalable optimization in the second phase.

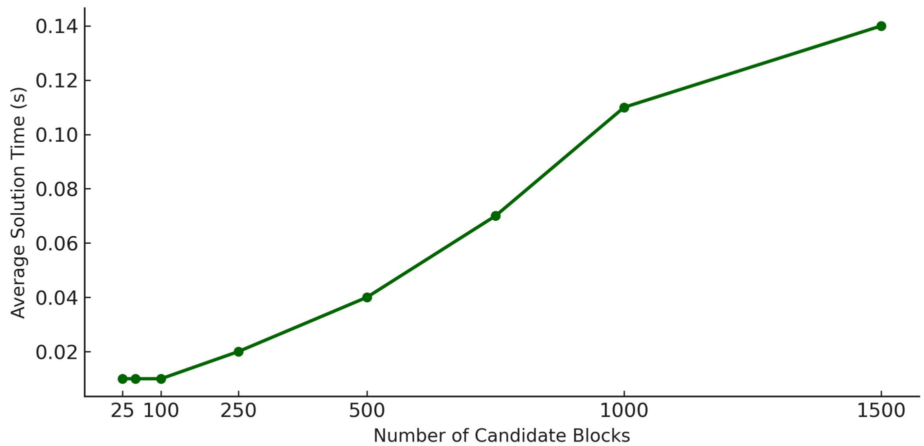

Figure 6.3 : R2: Average solution time (S) for different numbers of candidate blocks.

6.6 MODEL EVALUATION

To evaluate the proposed two-phase model, we compare it with a model introduced in [16], which formulates the day-ahead hourly bidding problem as a two-stage stochastic mixed-integer nonlinear programming model. In this reference framework, first-stage decisions determine the bid volumes, while second-stage decisions reflect the actual hourly dispatch under different price scenarios. In the hourly bidding model, the imbalance between committed and realized volumes is explicitly considered, with corresponding rewards and penalties determined based on participation in the balancing market. For comparison, all input parameters such as initial reservoir volumes, inflows, and operational constraints are considered identical in both models. Given that the hourly bidding model requires separate offers for each hour of the day, each block generated in our proposed approach is also designed to span a full 24-hour period. Moreover, the water usage cost has been added to the objective function of the hourly bidding model. The hourly bidding model also follows market rules, requiring offer curves to be non-decreasing with respect to price levels. Likewise, committed volumes are determined based on market-clearing prices using linear interpolation. Additionally, the same set of price scenarios used in the second phase of the block bidding model is applied to the hourly model to ensure that both methods are evaluated under identical market uncertainty.

After the market is cleared, the profit from each submitted block and the profit from the hourly bidding model are calculated for comparison purposes. In the profiled block bidding model, since at most one block can be accepted, the selected block will be the one with the highest offered price that does not exceed the market price. This ensures compatibility with market rules. Given that the hourly bidding model is a two-stage stochastic mixed-integer nonlinear program and that the case study involves five hydropower plants and six reservoirs, solving the model under a large number of scenarios presents computational challenges. Therefore, to enable a meaningful and tractable comparison between the two models, five representative price scenarios are considered. Due to the complexity of the hourly bidding formulation, start-up costs could not be incorporated in that model; hence, for consistency, start-up costs were also excluded from the block bidding model in this part of the evaluation.

The comparison results are summarized in Table 6.2, which includes multiple test cases evaluated with varying inputs and price scenarios on different days.

Table 6.2 : Comparison of hourly bidding and selected block profit

Case	**Number of Blocks**	**Number of Scenarios**	**Hourly Bidding Profit**	**Selected Block Profit**
Case 1	750	5	70,369	69,327
Case 2	750	5	144 ,060	147,883
Case 3	750	5	89 ,783	89,594
Case 4	750	5	54 ,069	55,380
Case 5	750	5	71 ,586	71,890

As shown in Table 6.2, the proposed method provides better results or results that are very close compared to the hourly bidding model. Block bids are particularly valuable for hydropower producers because they allow for offering energy over multiple consecutive hours with operationally feasible patterns. This is beneficial in systems with reservoir dependencies, start-up costs, or limited flexibility. Moreover, block bids contribute to more stable production schedules and better alignment with market prices under uncertainty. The model presented in this paper benefits from relaxing the binary variables in both phases, resulting in very short solution times. Therefore, it can be highly efficient and can be applied to complex and large-scale problems.

6.7 CONCLUSION

This paper presented a two-phase optimization framework for hydropower producers participating in the day-ahead electricity market using profile block bids organized in exclusive groups. The first phase generates a diverse set of feasible production blocks through a deterministic MINLP model that incorporates operational constraints, start-up costs, and opportunity costs. The second phase employs a two-stage stochastic program to select the most profitable combination of these blocks based on a set of market price scenarios, while respecting market design rules such as exclusivity within block groups. Computational results confirmed the model's effectiveness in producing high-quality bidding strategies with relatively low solution times. Additionally, a comparison with an hourly bidding strategy showed that the proposed method can achieve similar or improved profits while significantly reducing complexity. For future work, the opportunity cost formulation could be refined by incorporating more detailed seasonal patterns and inflow variability, allowing for a more accurate representation of water value dynamics over time. Additionally, while the current experiments were limited to five price scenarios due to the computational burden of the MINLP, evaluating the framework using a broader set of scenarios and a wider range of test cases would strengthen the robustness of the results. A possible extension would be to develop a model that integrates both hourly and profiled block bids for the day-ahead market, providing producers with a unified strategy to participate more effectively in electricity markets. Finally, further evaluation of the model under different system conditions—including reservoir connectivity, inflow uncertainty, and price volatility—would provide deeper insights into the practical benefits and limitations of profile-based bidding strategies.

CHAPTER VII
CONCLUSION

This chapter presents a general conclusion of the work carried out in this thesis. It also discusses their practical relevance, outlines the limitations encountered, and offers perspectives for future research to further improve the proposed approaches.

SUMMARY OF THE WORK

This thesis has explored short-term hydropower scheduling in deregulated electricity markets through the development and evaluation of three complementary optimization models. Each model addresses a different practical challenge faced by hydropower producers, and together they form a cohesive framework that combines operational realism with market participation strategies.

The first contribution introduced a nonlinear stochastic mixed-integer programming (MINLP) model for optimal hourly bidding in the day-ahead market under price uncertainty. The model incorporates key operational constraints and represents nonlinear relationships between discharge and reservoir volume using maximum power output surfaces. To solve the model efficiently, an iterative heuristic method was developed, where binary variables associated with turbine usage were progressively fixed across iterations. The model was validated against results from the SHOP model and demonstrated its ability to produce accurate and realistic bidding strategies. This contribution enhances the producer's ability to make informed and adaptive bidding decisions in the face of uncertain electricity prices.

The second contribution focused on short-term scheduling after market commitment, where the total production offered must be delivered hour by hour. A deterministic MINLP model was developed using maximum power output surfaces that link reservoir volume and turbine discharge to power generation. This formulation allowed for flexible representation of multiple turbine combinations while penalizing excessive startups. Three solution approaches were explored: an iterative heuristic, a genetic algorithm, and a hybrid method combining both. Results showed that the hybrid method provided the best compromise between solution quality and computational time, particularly for medium-sized systems, and effectively balanced production and startup costs. The ability to capture startup constraints and turbine configurations within realistic time frames makes this model particularly valuable for operational planning.

The third contribution introduced a two-phase optimization framework for block bidding based on profile blocks. In Phase 1, a deterministic nonlinear model was used to generate a diverse set of feasible profile blocks under operational constraints, including opportunity costs and startup costs. In Phase 2, a two-stage stochastic linear model selected the most profitable combination of profile blocks by evaluating their performance under multiple price scenarios close to delivery time. The proposed framework was applied to a realistic hydropower system in the Orkla river basin in central Norway, consisting of five power plants and six reservoirs. Numerical results demonstrated that the profile block bidding approach could outperform conventional hourly bidding by offering greater flexibility, better risk hedging, and more stable revenues under market uncertainty. This framework offers a scalable and practical strategy for producers aiming to participate effectively in markets that support block bids.

LIMITATIONS AND PERSPECTIVES

While the results across all three models are promising, certain limitations must be acknowledged. In the first contribution, the assumption that the producer is a price-taker may not always be valid, especially for large producers whose offers can influence market prices. Additionally, the use of fixed terminal reservoir volumes across all scenarios may limit flexibility under volatile inflow conditions. The binary fixing heuristic, although efficient, may overlook more optimal configurations that emerge later in the solution process.

In the second contribution, although the output surface approach simplifies the nonlinear relationship between volume, discharge, and power, it also introduces approximation errors that might limit accuracy in extreme operating conditions. The scalability of the hybrid solution method has only been tested on systems of moderate complexity; applying it to larger, interconnected systems with cascading reservoirs would be a meaningful next step.

Regarding the third contribution, the independence of profile generation from market scenarios in Phase 1 could limit adaptability under certain market conditions. Moreover, the stochastic model assumes fixed scenario probabilities, which may not reflect real-time market updates or forecasting errors. Another limitation is that the framework has only been validated on one case study; broader application to more diverse systems and market settings is necessary to generalize its utility.

Future research could address these limitations in several ways. First, incorporating joint uncertainty in both prices and water inflows would provide a more comprehensive and realistic planning model. Developing advanced metaheuristic algorithms—such as Simulated Annealing, Ant Colony Optimization, or hybrid evolutionary methods—could help solve complex non-convex formulations more efficiently. In block bidding models, refining the opportunity cost function to include seasonal water value variations or long-term strategic considerations could improve bidding accuracy. In addition, integrating hourly and block bidding into a unified decision-making framework would better reflect current market mechanisms and give producers greater flexibility. Finally, applying the proposed methods to large-scale systems with multiple hydropower stations, strong spatial dependencies, and high price volatility would provide further insight into their practical relevance and scalability.

PRACTICAL RELEVANCE AND CREDIBILITY OF THE RESULTS

The models developed in this thesis were designed with a focus on practical use, reflecting real operating conditions and market requirements. They were tested on case studies using verified data. The price scenarios used in the first and third objectives were provided directly by data suppliers, and their quality was confirmed by them.

One of the main aims of this work was to show that nonlinear models can be applied in the day-ahead electricity market. Research in this area is still limited, but the findings of this thesis show that such models can work in practice. They converge within short solving times and provide solutions that are reliable and consistent, which makes them suitable for real bidding situations.

A common difficulty with stochastic models is the computational burden, especially when many scenarios and binary variables are included. In this work, several strategies were applied to keep the models solvable. For example, maximum output surfaces were used instead of modeling each turbine separately, which simplified the problem while still capturing the nonlinear effects. In the heuristic methods, binary variables were fixed step by step in an iterative process, which helped the solver converge faster. In the first objective, the property of total unimodularity was satisfied only in part of the solving process, which allowed some of the binary variables to be relaxed in order to save computation time while keeping the quality of the solutions. In the third objective, since this property held in both phases, all binary variables could be relaxed without losing optimality.

The proposed models showed fast performance and reached solutions within short computation times. In real market settings, producers often reduce the number of scenarios to make the models manageable, and the methods developed here help keep the solving process efficient in such cases.

The credibility of the results was also strengthened by validation against SHOP, a tool widely used in the hydropower industry, which confirmed the practical accuracy of the proposed approaches.

In the third objective, an opportunity cost term was *explicitly included* in the formulation and modeled as a linear function for tractability. This term can be refined in future work to reflect seasonal water values or more detailed water value dynamics. Future research could also include joint uncertainty in inflows and prices.

This thesis shows that nonlinear optimization models can also be applied in the day-ahead electricity market. They are computationally efficient, validated with real data, and able to provide reliable solutions within short solving times. At the same time, the proposed models have clear potential for further development and improvement in future research.

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