Development of a damage simulator for the probabilistic assessment of building vulnerability in subsidence areas

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Abstract:

The extraction of ore and minerals by underground mining often causes ground subsidence phenomena and may result in severe damage to buildings. Risk analysis in subsidence regions requires the assessment of both the hazards to and vulnerability of nearby buildings. However, many uncertainties exist and this assessment and its representation as well are still a complex objective. For this purpose a damage simulation tool is developed to investigate hazard and vulnerability under several possible scenarios of mining subsidence in which a large number of buildings may be affected. Ground movements assessment is based on the influence function method, and building damage is estimated using vulnerability functions.

A case study is presented to illustrate the different results given by the damage simulator. Uncertainties about the collapsed zone of the mine and influence angles lead to the definition of different possible scenarios. A relative occurrence probability is then defined to implement a probabilistic approach to the hazard and vulnerability assessments. Different results, more or less synthetics, can then be obtained to assess both hazard and vulnerability over the exposed city. These results are compared and the maximal horizontal ground strains and the mean damage appear to be the most effective and relevant way to address the question. A final ranking based on scoring is then provided.

KEYWORDS: hazard analysis, building vulnerability, mining subsidence, damage, probabilistic approaches.

1. Introduction

Risk assessment and mitigation is a key concern for cities affected by natural hazards. These tasks require both an accurate prediction of the hazard and a careful evaluation of building vulnerability in spite of the existence of several uncertainties. In technical settings, the hazard can be quantitatively described as “the likely frequency of occurrence of different intensities for different areas” [1] and vulnerability as “the conditions determined by physical, social, economic, and environmental factors or processes, which increase the susceptibility of a community to the impact of hazards” [2]. However, the term "vulnerability" is frequently used in the strict sense of building strength.

In recent years, different risk assessment methodologies have been developed and incorporated in a considerable number of different software ([3], [4], [5], [6]). Such softwares and methodologies may significantly improve the assessment and the visualization of both hazard and vulnerability at a city scale. A first conclusion is that such approaches are actually seldom developed in the context of mining subsidence hazard. Recently, Malinowska and Hejmanowski ([7]) proposed a risk assessment method for mining subsidence zones with GIS data. This method represents an advance in risk assessment techniques for mining subsidence but is not comparable with existing methods for risk assessment associated with other natural hazards. Firstly it uses an empirical building damage assessment instead of vulnerability functions mainly used otherwise. Secondly this method does not consider the uncertainties in the two main parameters of the risk assessment, namely, building damage and hazard assessment, while this objective is crucial in this paper.

The objective of this paper consists into the development of a probabilistic approach of the building damage assessment and the analysis of the possible issues that may help for the risk assessment. This first leads to develop software named mining subsidence damage simulator (MSDS) in the following. This paper focuses on the influence of uncertainties, which is a key point for risk management and may affect both the building vulnerability and the hazard assessment. Uncertainties about vulnerability are first taken into account through vulnerability curves, which are based on the definition of a building typology, the use of a hazard intensity criterion and the definition of a damage scale.

Vulnerability curves are relationships between the damage mean value μ_D for a given type of building and the value of the hazard intensity. They are developed for each building type, and they allow a quick and realistic damage assessment of all the buildings that are grouped into the same type. Vulnerability functions can be calculated with the fragility curves and Eq. (1) [8].

$$\mu_D = \sum P_k . D_k \qquad (1)$$

Where μ_D is the mean damage for a particular value of hazard intensity, D_k the damage level between 0 to 5 for a five levels damage scale ($D_0 = 0$ for no damage and $D_5 = 5$ for very severe damage) and P_k is the probability of a damage level D_k.

The use of vulnerability function is now a common way to assess building damage for many natural hazards ([3], [8], [9]). However, they require knowing the value of the hazard intensity, whereas this is also an uncertain parameter.

From a theoretical point of view, if uncertainties on hazard may be assessed by defining different possible scenarios with different intensities and probabilities, then risk management requires to address the building damage assessment by considering the whole possible scenarios. Methods used to define these scenarios may be specific for each kind of hazard. In the following, a methodology based on both expertise and computations is developed in the field of mining subsidence hazard to assess a set of scenarios. The MSDS is applied to this set of scenarios in order to develop a probabilistic assessment of the vulnerability. Different strategies are investigated to synthetize the results.

This paper is organized into 3 sections. First section is a description of the mining subsidence hazard and methods used in the MSDS in order to assess the building damage in relation to the characteristics of both the underground mine, overburden and buildings. Second section focuses on uncertainties and more specifically on the description of the methods used to define a set of realistic scenarios. Third section is the development of the probabilistic assessment of the vulnerability taking into account all uncertainties. A case study is investigated through these different sections.

2. Development of the mining subsidence damage simulator (MSDS)

2.1. Underground mines and subsidence

Underground mining operations cause ground subsidence. This phenomenon leads to horizontal and vertical ground movements, which lead to deformation of and damage to buildings in undermined urban areas (

Fig. 1). The maximum vertical displacement may reach several meters [10]. This vertical displacement is accompanied by horizontal ground strains, ground curvature and slope, the three types of ground movements that may cause structural damage. Depending on the mining extraction method used, whether it is longwall or rooms and pillars with or without caving of pillars, subsidence can be planned. In some cases it can also be unexpected a long time after the extraction. In all cases, the prediction of building damage is necessary when subsidence is expected in an urbanized area [11]. This paper mainly focuses on mining area with abandoned rooms and pillars mines that may induce unexpected subsidence.

Many countries are concerned with mining-subsidence-induced damage (for example, England, the United States, Poland, Germany, France, South Africa, India, China and etc.). Therefore, different methods have been developed to assess ground movement: empirical ([12], [13]) or analytical ([14], [15]). The most important parameter used to quantify the subsidence intensity and assess the building damage is the horizontal ground strain. These two kinds of methods may be used to develop vulnerability curves for different buildings types ([16], [17]). These curves will be used in the following.

2.2. Principles of the MSDS

The MSDS aims to use a geographical information system (GIS) for the representation and the spatial localization of both the buildings and underground mines. Its objective is to assess and represent building damage for any specific mining subsidence. The MSDS is based on a very simple scheme illustrated in Fig. 2 with the following input and methods:

a) a method to predict the subsidence parameters over a geographical area due to the collapse of a mine or part of it (vertical subsidence, curvature and horizontal ground strain). As Malinowska and Hejmanowski [7], the influence function method is chosen because it allows realistic assessments for any shape of the underground mine [16]. This method is based on the superimposition principle [10] and uses a set of parameters that must be adjusted in relation to any specific case study. In the perspective of the development of a probabilistic approach, these parameters can be assumed uncertain;

b) vulnerability functions to assess building damage due to mining subsidence, based on Saeidi et al. 2009 and 2012 ([13], [17]). For each case study, this requires to classify each building into a given typology and to develop specific vulnerability curves. In the perspective of the development of a probabilistic approach, the vulnerability functions may also be assumed uncertain;

c) a set of realistic subsidence scenarios in relation to the characteristics of the underground mines. Each scenario corresponds to a mining area that is assumed to collapse. In the perspective of the development of a probabilistic approach, the exact shape of the collapse mine can be assumed uncertain;

d) the use of a building database that consists of a digitized map of the studied area with the exact coordinates of all the buildings and the mechanical and geometrical characteristics of each building (building typology);

e) a calculation module, developed with Mathematica® ([18]), that enables the damage of each building to be assessed depending on its characteristics (vulnerability or fragility curve) and the local ground movements (Fig. 2);

The following section describes the methods used to predict the subsidence parameters and the vulnerability functions. Then, a case study is investigated with a first analysis that considers a deterministic hazard, i.e., a given subsidence event. Finally, a second analysis is performed to take into account uncertainties associated with hazard assessment, i.e., a set of possible subsidence, and to develop a probabilistic assessment of building vulnerability.

3. Methods used to predict the subsidence hazard and the building vulnerability

3.1. Subsidence hazard

There are several methods used in mining engineering for the prediction of the subsidence ground movements. These methods can be classified as empirical, semi-empirical, analytical or numerical A large description of these methods can be founded in Whittaker and Reddish [10].

Numerical methods make use of various methods like the finite elements method, the distinct elements method or the finite differences method. These methods can be very accurate when validated, but their application at a specific site and or in a certain context is highly dependent on the available data regarding the local geology, the mechanical properties of the overburden and sub-surface rock/soil. Moreover calculating a three dimensional prediction of the subsidence may require a large computational effort [19].

Graphical methods are derived from analysing an extensive field database collected over many years from mining subsidence in one country. A disadvantage of these methods is that they are developed in relation to a specific context and cannot be used with accuracy in other contexts. A well-known example has been developed by the NCB ([12]), which has provided several abacuses that can be used to predict subsidence for simple geometry mines (rectangular).

The profile function methods are based on mathematical functions that have been obtained by a curve fitting procedure to match the predicted profile with observed profiles [10]. Many profile functions are available for subsidence prediction [10]. These methods suffer from the same disadvantage as the graphical methods: they can be used only in specific contexts [7]. Another disadvantage is that these methods are developed to predict a two dimensional subsidence profile and are not intended to predict the whole three dimensional subsidence.

Influence function methods (IFMs) were developed by Ren et al. ([20]) and are used extensively ([10]), ([21], [22]) to predict mining subsidence. They are based on the superposition principle and address the displacements induced by a subsidence at a given point as the sum of the displacements induced by the subsidence of elementary mining units. The superposition theory is only valid for purely linear elastic phenomena, while important inelastic and nonlinear phenomena actually occur during subsidence ([16]). Consequently, different coefficients are suggested to adjust the results of the superposition ([10] ,[23]).

Nevertheless, IFMs present several advantages compared to other methods for the three dimensional prediction of subsidence. First, these methods can be used with any type of mine geometry; empirical and semi-empirical methods are restricted to simple geometries. Secondly, these methods can be used to simultaneously assess vertical and horizontal ground movements induced by the subsidence at each point of the surface. In particular, the horizontal ground strain can be calculated everywhere and then used to assess the building damage ([13], [17]). This method has then be chosen for implementation in MSDS and is further explained in the next section

3.2. Influence function method

This method is based on the superposition principle and addresses the displacements induced by a subsidence at a given point as the sum of the displacements induced by the subsidence of elementary mining units. For example, if s_1 and s_2 are the vertical subsidence of surface points caused by the collapse of the surfaces A_1 and A_2, respectively, of an underground operation, then the subsidence caused by the collapse of the two surfaces A_1+A_2 is s_1+s_2.

The elementary subsidence dS_z at a given point P in the surface, caused by an elementary mining surface dA at depth H to be extracted is calculated with Eq. (2).

$$dS_z = S_{max} \times K_z(r, \gamma) \times dA \quad (2)$$

where $S_{\max}$ is the maximum value of subsidence that can be observed for a critical and super critical case (i.e., for mines wider than $2H\tan(\gamma)$, where H is the depth and γ the influence angle); $K_z(r, \gamma)$ is the influence function, where r is the radial distance between the surface dA and the surface point under consideration and γ is the influence angle (Fig. 3).

The final vertical subsidence at a given point P on the surface can then be estimated by integrating Eq. (2) over the mine panel surface (A) (Eq. (3)).

$$S_z = S_{max} \iint_A K_z \times dA \qquad (3)$$

Numerous influence functions (IFs) exist in the literature that are either derived from empirical observations or based on theoretical assumptions [24]. All IFs have the same aim: accurately model the subsidence of the ground surface. Nevertheless, they show significant differences in ground profiles because of geological setting variations in each mining field. The selection of a particular IF, therefore, needs to be validated by a comparison with previous existing subsidence to take into account the geological conditions of the studied site [16].

In this analysis the Beyer influence function [10] is used because its application is validated in the Lorraine basin region with back analysis of the results of happened subsidence [23].

The influence function method (IFM) can be used to calculate the horizontal displacement (U_{xy}) of each point at the surface and, consequently, the horizontal ground strain. The horizontal displacement is first calculated based on focal point theory, which assumes that each extraction element dA (Fig. 4 a) attracts a surface point P that moves towards it by dU. As shown in Fig. 4 a, the vector dU can be represented by two orthogonal components, dS_z and dU_{xy}. dS_z is the vertical subsidence, and dU_{xy} is the horizontal radial displacement resulting from the extraction of element dA. Therefore, the horizontal radial displacement is calculated with Eq. (4) [10].

$$dU_{xy} = dS_z \times Tan\xi \qquad (4)$$

Where ξ is the angle between the vertical axis and the line joining the surface point P with the extraction element dA (Fig. 4 a).

By considering α, the angle between the radial horizontal displacement vector (U_{xy}) and the X-axis, the two components of the radial displacement, dU_x and dU_y, can be calculated from Eq. (5) (Fig. 4 b; Whittaker and Reddish, 1989).

$$\begin{aligned} dU_x &= dU_{xy} \times cos\,\alpha \quad \Rightarrow \quad U_x = \sum dU_x \\ dU_y &= dU_{xy} \times sin\,\alpha \quad \Rightarrow \quad U_y = \sum dU_y \end{aligned} \qquad (5)$$

After calculating the horizontal ground displacement in the X and Y directions (U_x, U_y), continuum mechanics theory is used to calculate the horizontal ground strain (Eq. (6)) in the x and y directions. Then, the maximal horizontal ground strain is calculated with Eq. (7).

$$\varepsilon_x = \frac{\delta U_x}{\delta x}$$
$$\varepsilon_y = \frac{\delta U_y}{\delta y} \tag{6}$$
$$\gamma_{xy} = \left(\frac{\delta U_x}{\delta y} + \frac{\delta U_y}{\delta x}\right)$$

$$\varepsilon_{min}^{max} = \frac{\varepsilon_x+\varepsilon_y}{2} \pm \sqrt{\left(\frac{\varepsilon_x-\varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2} \tag{7}$$

This method has been implemented in the damage simulator so that the horizontal ground strain can be assessed anywhere at the surface for any mining geometry.

The input data for this part of the damage simulator are the maximum value of subsidence S_{max} that can be observed for a critical and super critical case, the choice of the influence function, the geometry and depth of the mine.

The outputs of this part of the simulator are the contour of vertical subsidence, horizontal displacement and horizontal ground strain. When this method is associated with a GIS, it is possible to assess the horizontal ground strain in the vicinity of any building. The next section presents the methods used to assess the building vulnerability.

3.3. Building vulnerability

Building damage assessment in mining subsidence hazard areas is a key point for risk management. The main obstacle is that existing methods, developed in different countries, are more appropriate for the study of single buildings than for large urban areas. These methods could be divided into three groups: empirical ([12], [13], [25]), analytical ([14], [15], [26], [27], [28], [29],) or numerical ([19], [30], [31], [32], [33], [34], [35]). A large description of these methods can be founded in Saeidi et al. ([13], [17]).

All of these methods make a deterministic evaluation of the damage, but experience shows that similar adjacent buildings affected by the same subsidence may suffer different damage. The problem of the uncertainties for building damage assessment is addressed in other fields of risk analysis, such as seismic engineering [3, 5, 8]. It is based on the use of vulnerability and fragility curves to assess the mean amount of damage and the damage distribution of all buildings with similar characteristics in relation to the event intensity. This approach has proven to be a good compromise between accuracy of the results and necessary investment for the studies (cost and duration) ([36], [37]).

Fragility curves provide the probability of reaching or exceeding a given damage state as a function of the event intensity. These Curves follow a lognormal function. Vulnerability curves are relationships between the mean amount of damage for a given type of building and the value of the event intensity. These Curves follow a hyperbolic tangent equation ([36], [37], [38]).

In the case of subsidence, the horizontal ground strain is used as intensity criteria, because it is the most shared parameters into the different building damage assessment methods. Moreover, curvature and horizontal ground strain display a homothetic variation along the subsidence profile. Specific developments may then be provided so that the damage predicted with a vulnerability function, for a given value of the horizontal ground, takes also into account the curvature [17].

A tangent hyperbolic function is used to model the vulnerability functions and provide continuous values for the damage mean (Eq. (8)).

$$\mu_D(\varepsilon) = a[b + tanh(c \cdot \varepsilon + d)] \quad (8)$$

Where $\mu_D(\varepsilon)$ is the damage mean value for a value ε of the hazard intensity, and *a*, *b*, *c* and *d* four coefficients that must be determined for each building type.

Vulnerability functions developed in [13] and [17] are used in the MSDS. They are normalized so that $\mu_D(\varepsilon)$ is between 0 and 1.

4. Data relevant to the case study

The town of Joeuf is located in the iron-ore basin in Lorraine, in north-east of France. Joeuf has more than 1,500 buildings and more than 7,000 inhabitants. The town sits atop numerous underground iron mines that were exploited beneath the entire city at a depth of approximately 90 m and an ore thickness of up to 20 m ([39], [16]). In some areas, there are three superimposed underground layers. The extraction system is the room and pillar mining method. The first set of required data concern all information about the mines relevant to the prediction of the ground movements associated with the collapse of a sector of the mine.

Many districts are workers' housing sets with similar building types that consist of jointed masonry buildings with 1 or 2 floors. Most of the buildings were constructed between 1870 and 1930 ([16]). Some districts also contain more recent buildings with concrete materials that represent higher-quality construction than the older buildings. The database includes all information about the buildings that is needed for the development and use of adapted vulnerability functions (e.g., length, height, materials and reinforcements).

4.1. Mining data

The mine-related data consists of the geographical coordinates and the characteristics of each mining panel that may collapse due to ageing and flooding. The identification of these panels is based on mechanical and geometrical criteria, back analysis and expert judgment.

Under the town of Joeuf, three sub-horizontal iron layers were exploited (Fig. 5): the brown layer (deepest), the grey layer, and the layers S_2 and S_3 (shallowest) extracted simultaneously. The mining method employed was the rooms and pillars method with different extraction ratios: 21% for the brown layer, 35% for the grey layer and 45% for the S_2/S_3 layers ([16]). Because of the low extraction ratios for the brown layer, only the grey and the S_2/S_3 layers are expected to collapse. The depth of the mines is variable because of the topography of the ground surface. However, the city is lying into a small valley and the depth is then assumed constant over the city. Fig. 5 a presents schematic vertical section of the mine under the Joeuf town [16].

To delineate the different polygons of mines that may cause mining subsidence on the surface, we looked for areas characterized by high extraction ratios and/or small pillars and/or pillars with a heterogeneous geometry, bordered areas that have not been undermined or are more robust (i.e., with small local extraction ratios and/or large and regular pillars). A first deterministic analysis leads to assume that if a collapse begins within a polygon, the collapse can extend up to the boundaries of the polygon, where stronger or intact ground exists. For instance, Fig. 5 b, c and d show the five initial polygons defined within the grey and S_2/S_3 layers and mining data for each polygon are synthetized in Table 1. Value of the maximal subsidence S_{max} is calculated with Eq. (9), with an empirical coefficient k_s = 0.5 based on back analysis of historical subsidence.

$$S_{max} = k_s \, w \, \tau \qquad (9)$$

where w is the mining opening, τ the extraction ratio (between 0 and 1) and k_s an empirical parameter.

However, some uncertainties exist and we can assume that the collapse may stop before reaching the polygon boundaries. In that case randomly reduced polygons can be considered with the borderer included between two deterministic limits: the initial polygon borderer and a homothetic reduced polygon by a 0.5 scale factor. Fig. 6 illustrates the method used with the initial and 0.5 scale reduced polygon for the polygon 3 and a set of 10 random polygons.

The influence angle parameter γ mostly depends on the overburden geology and its geotechnical characteristics. It may also depend on the mining exploitation method, and its value is typically between 5 and 40° [16]. Influence angles may also vary around a mine panel. This is the case in the French Lorraine iron-ore basin, where the influence angle depends on the nature of the ground beyond the boundary of each edges of the mine polygon [39]. Different influence angles are then considered with uncertainties about +/- 10° (Table 2).

4.2. The relative occurrence probability of the mining subsidence scenarios

Assessing the occurrence probability of each scenario is a particularly complex problem. No method exists to rigorously assess this probability, and expertise is generally required. However, the analysis of different geometrical and mechanical parameters can be used to order the scenarios from the least to the most probable.

First, a subsidence is more likely to occur when the safety factor, calculated from the induced stresses and the compressive strength of pillars, decreases (Eq. (10)).

$$SF = \frac{\sigma_c}{\sigma_P} \tag{10}$$

Where σ_P denotes the compressive stress on the pillars, and σ_C the compressive strength of the pillars.

The compressive stress on the pillars can be assessed with the tributary area model, which assumes a uniform distribution of the weight of the overburden over the pillars (Eq. (11)).

$$\sigma_P = \frac{\rho * g * H}{1-\tau} \tag{11}$$

Where ρ is the unit mass of the overburden in [kg/m^3], H is the thickness of the overburden in [m] and τ is the extraction ratio between (total extraction ratio) and 1 (no extraction)[40].

If the rock compressive strength is assumed to be constant over the mine, then the safety factor (SF) only depends on the vertical stresses on the pillars (Eq. (16); [40]). However, the parameters used to evaluate these stresses are still uncertain (mainly the geometry of pillars), and a Monte Carlo simulation is particularly helpful to assess the safety factor as a probabilistic variable rather than a fixed value. In the specific case of the iron-ore field, Cauvin *et al.* (2009) [41] showed that the safety factor can be modeled with a normal distribution whose mathematical expectation is calculated with a compressive rock strength of 7.5 MPa and whose standard deviation is approximately 0.3.

For the considered mining polygons and data in Table 1, we obtain a mean and standard deviation {m, s} of the safety factor of {2.05, 0.082} for polygons 1, 2, 3 and {1.97, 0.0788} for polygons 4 and 5. A collapse is expected to occur if the safety factor is less than one. The cumulative density function is then used to assess the probability of collapse $p(S_0)$ for an elementary area S_0: 6.1 10^{-4} for {m, s} = {2.05, 0.082} and 2.3 10^{-4} for {m, s} = {1.97, 0.0788}.

The second parameter used to assess the relative probability is the area of each polygon. If two polygons have the same safety factor but one polygon is twice as large as the other, then a collapse is more likely to occur in the larger of the two polygons. If $p(S_0)$ is the collapse probability for an elementary area S_0, then the collapse probability $p(S_k)$ of a larger area S_k can be calculated with Eq. (12).

$$p(S_k) = 1 - (1 - p(S_0))^{S_k/S_0} \quad (12)$$

The application of Eq. (12) to the six considered polygons requires the identification of the elementary area S_0. Because the objective is to assess a relative probability of collapse, the choice of this area has no influence upon the final results, and the area of the smallest polygon (polygon 7) is chosen for S_0 ($S_0 = S_7$).

Finally, the collapse probabilities $p(S_k)$ of the six polygons are normalized to obtain the relative probability $p_R(S_k)$ so that the sum of the six values equals one (Table 3).

4.3. Building database

The building database collects the geographical coordinates and some characteristics of 1102 buildings. The geographical coordinates are necessary for calculating the exact value of the ground movements due to the subsidence in the vicinity of each building. The building characteristics are necessary to define the building typology and to develop appropriate vulnerability functions (e.g., length, height, materials and reinforcements).

Based on a detailed analysis of the existing buildings in the town, a total of three main building types are defined. Most of the buildings (89%) consist of unreinforced masonry buildings (URM), 9% of them consist of reinforced masonry buildings (RM) and 2% of reinforced concrete buildings (RC). This pattern is a consequence of the historical urban development involving many workers' housing complexes (Fig. 7).

Three vulnerability curves are then considered. First for unreinforced masonry buildings, second for reinforced masonry building and third for concrete buildings. Bases on uncertainties about the values of parameters a, b, c and d (see Eq. (8)) observed for unreinforced masonry buildings], an equivalent uncertainty is considered for all the three curves. Parameter c is fixed for each function, while parameter d is considered uncertain with a uniform distribution. Parameters a and b are calculated in order to respect two conditions (Eq. (13)). These conditions correspond to a zero damage for a null horizontal ground strain and an ultimate damage for horizontal strains greater than 10. For the 3 building types, vulnerability curves are then included between two limit curves denoted + and – in Fig. 8.

$$\begin{cases} \boldsymbol{\mu_D(0) = 0} \\ \boldsymbol{\mu_D(10) = 1} \end{cases} \tag{13}$$

5. Deterministic vs. probabilistic results of the MSDS

In the next sections, results of the MSDS are compared in order to show how uncertainties may have an influence on the hazard and vulnerability assessment over the city. Results are differently displayed from the most exhaustive to the most synthetic. The most exhaustive corresponds to a colorized map that displays the hazard intensity (horizontal ground strain) or damage for each building. A more synthetic result is the histogram that shows the number of buildings into each hazard or damage classe. The most synthetic results give the mean value and standard deviation of hazard or damage for buildings in the city. In that case, two values are considered. The first value considers all buildings, while the second value consider only the buildings affected by the subsidence (i.e with a damage greater than 0.01 or a hazard intensity greater than 0.1 mm/m).

5.1. Results associated with the collapse of a given mining polygon

Two results can be showed: the hazard intensity as the horizontal strain in the vicinity of each building and the vulnerability with building damage (Fig. 9, Fig. 10 and Table 5). Building damage is then calculated by combining the horizontal ground strain assessed at the center of each building and the vulnerability functions. Results of the building damage calculation are then very close to the hazard calculations (Fig. 9); buildings are logically more damaged above the border of the polygon, where the horizontal ground strain is the greatest.

A first comparison between each polygon immediately shows (Fig.10) that polygon 3 seems the most critical with a greater number of affected buildings. The damage mean value for the entire city is then maximal (Table 5). However, as shown on Table 5, polygon 2 becomes more critical when only the damaged buildings are considered. Histograms (Fig. 10) explain the difference between the two cases. Histograms of hazard intensity display a global hyperbolic shape with a large number of buildings faintly or not affected and a small number strongly affected. In the contrary, the histograms of damages display two peaks both for small and severe damage.

5.2. Building damage associated with different possible scenarios of mining subsidence

The aim of this section is to consider now different possible scenarios with uncertainties. In the following, for each of the five deterministic polygons, 10 scenarios are considered with a variable polygon shape (Fig. 6), a variable influence angle (Table 2) and variable vulnerability functions (Table 4). A uniform distribution is chosen for each variable parameter. This gives a final number of 50 scenarios for which both the horizontal ground strain and damage is calculated for each building. The main issue is now to define the hazard intensities and the building damage when several scenarios may occur. In other words, how to synthetize these fifty results in order to get relevant information for the risk management?

A first solution is to consider all the mining scenarios as deterministic and to define the hazard intensity and the damage category of each building as the maximal value obtained from any of the five scenarios (Eq. (14)).

$$\begin{matrix} Intensity1 = Max(\varepsilon_k) \\ Damage1 = Max(D_k) \end{matrix} \quad for\ k = 1\ to\ N \tag{14}$$

Where N is the number of scenarios ($N=50$ in this example), and ε_k and D_k are the horizontal ground strain in the center of a given building and its damage, respectively, associated with scenario k. The results for the whole town are shown in Fig. 11. Results regarding the building damage is not very useful since quite the whole city is concerned. Results regarding the intensity appear more interesting with a quite uniform distribution of buildings into the different classes of horizontal ground strain. This way then appears more useful to categorized the city and identify areas that are the most concerned. However, this assessment method is highly conservative regarding the hazard intensity and building damage since only one scenario is expected to occur and not necessarily the worst among those studied.

A second solution consists in calculating a mathematical expectation and a standard deviation of both the hazard intensity ($E(\varepsilon)$ and $S(\varepsilon)$) and the building damage ($E(D)$ and $S(D)$) based, first on an assumption of equiprobability or on the relative probability (se 4.2) of each scenario (Eq. (15) and (16)).

$$\begin{matrix} E(\varepsilon) = \sum_{k=1}^{N} P_k . \varepsilon_k \\ S(\varepsilon) = \sqrt{\sum_{k=1}^{N} P_k . (\varepsilon_k - E(\varepsilon)))^2} \end{matrix} \tag{15}$$

$$\begin{matrix} E(D) = \sum_{k=1}^{N} P_k . D_k \\ S(D) = \sqrt{\sum_{k=1}^{N} P_k . (D_k - E(D)))^2} \end{matrix} \tag{16}$$

Where P_k is the relative occurrence probability of the mining subsidence scenario k, ε_k is the horizontal ground strain at a given building and D_k is the damage to the building.

Fig. 12 obtained under the assumption of equiprobable scenarios, seems not very useful for the risk assessment. Mean values of the horizontal ground strain are small with the greatest values around polygon 2 in the south of the city (Fig. 5). However the damage mean is greater in another part of the city. This difference is the consequence of non-linearity between damage and the horizontal ground strain.

A more interesting result is shown on Fig. 13 where all scenarios are not assumed equiprobable. The mean values of the horizontal ground strain and damage are then a little bit more comparable with similar locations of high impacted buildings. Compared to Fig. 12, this better correlation between the two results can be explained since one scenario is significantly more probable (polygon 4) and the most impacted area is closed to the borders of this polygon. Finally, Fig. 13 B is undoubtedly a relevant way for identifying the most impacted area.

A last possibility is to examine the standard deviation of damage. This can be used as an indicator of the confident level of the assessment. A small standard deviation means that the prediction is very confident while a large value means the contrary. Standard deviation of building damage versus the mean value is plotted on Fig. 14 in the case of non equiprobable scenarios. It can be observed a strong correlation between the two. Relation is not linear but the rough estimate of the standard deviation is about the value of the mean. Because of this strong correlation, the standard deviation is not an additional relevant parameter to classified city areas in the perspective of the risk mitigation.

As a consequence, two basic parameters are found to be interesting and not similar to identify the most impacted areas in the city. The first is the maximal value of the horizontal ground strain (Fig. 13 A) which corresponds to a conservative approach of the maximum hazard that may affect a building. The second is the damage mean value when scenarios are not assumed equiprobable, which corresponds to a reasonable assessment of what is the most probable to occur taking into account all uncertainties.

Therefore, we can propose a synthesis using these two most interesting parameters where a score is affected to each building in relation to both the two results. The final score is then the sum of two values between 0 and 4 and can finally range between 0 and 8. Application to the case study shows here that the maximal value of the final score is 4. As shown on Fig. 15, the map of the final ranking is roughly consistent with the two others previously shown. However, the comparison of histograms show that significant differences still exist. The limit values of the horizontal strain or the damage mean value in Table 6 can of course be discussed and modified.

6. Conclusion

A damage simulator has been developed to study the vulnerability of urban areas subjected to mining hazards. The simulator is based on two needs: first, to assess the ground movements associated with the collapse of any given underground mine; second, to assess the building damage associated with any given ground movement.

This simulator can be used to investigate different mining subsidence scenarios and to assess and categories both the hazard potential and the damage estimates according to established definitions of the hazard and the damage. For example, the simulator can calculate the maximal value of the horizontal ground strain or building damage, the mathematical expectation when considering equally probable scenarios, or the mathematical expectation when considering different occurrence probabilities for each scenario.

The last case corresponds to the probabilistic approach, for which the calculation of the occurrence probability of each scenario has raised many questions and for which a solution has been proposed in this study based on the safety factor and the area of each mine.

The results obtained with the developed method show that the probabilistic approach performs better than the other two considered methods. The first method, based on the maximum values, leads to a conservative assessment of the hazard and vulnerability, whereas the second method, which is based on equally probable scenarios, smoothes the results.

Finally, with the damage simulator, the mathematical expectation and standard deviation of the probability of each damage category for each building can be computed along with other information that may aid in the assessment of building vulnerability.

In conclusion, the methods incorporated in the developed damage simulator account for uncertainties in both hazard and vulnerability to provide risk assessment and mitigation. The simulator is presently applied to problems relating to mining subsidence hazards, and it is under development for other hazards for which a similar methodology can be followed provided that the hazard or vulnerability assessments involve uncertainties.

7. References

[1] UN/ISDR. Terminology on disaster risk reduction. Genova, Switzerland, United Nations International Strategy for Disaster Reduction (UNISDR); 2009.

[2] UN/ISDR. Living with Risk: A global review of disaster reduction initiatives. Genova, Switzerland, United Nations International Strategy for Disaster Reduction (UNISDR); 2004.

[3] Federal Emergency Management Agency (FEMA. HAZUS®MH MR2: Multi-hazard Loss Estimation Methodology Maintenance Release2(MR2).Federal Emergency Management Agency, Washington DC, United state, available form: http://www.fema.gov/library/viewRecord.do?id=2896S (accessed 03.11.09); 2006.

[4] Crowley H, Pinho R, Bommer JJ. A Probabilistic Displacement-based Vulnerability Assessment Procedure for Earthquake Loss Estimation. Bulletin of Earthquake Engineering. 2004;2:173-219.

[5] McGuire RK. Seismic Hazard and risk analysis. Oakland, CA: Earthquake Engineering research Institute; 2004.

[6] Oliveira, CS, Roca, A, Goula, X. Assessing and managing earthquake risk: geo-scientific and engineering knowledge for earthquake risk mitigation: developments, tools, techniques. Ed. Springer; 2006.

[7] Malinowska A, Hejmanowski R. Building damage risk assessment on mining terrains in Poland with GIS application. International Journal of Rock Mechanics and Mining Sciences. 2010;47: 238-45.

[8] Lagomarsino S, Giovinazzi S. Macroseismic and mechanical models for the vulnerability and damage assessment of current buildings. Earthquake Engineering. 2006; 4: 415–443.

[9] Kircher CA, Nassar AA, Kustu O, Holmes WT. Development of Building Damage Functions for Earthquake Loss Estimation. Earthquake Spectra. 1997;13:663-82.

[10] Whittaker BN, Reddish DJ. Subsidence: Occurrence, Prediction, Control. Elsevier Editions; 1989.

[11] Deck O, Verdel T, Salmon R. Vulnerability Assessment of Mining Subsidence Hazards. Risk Analysis. 2009; 29: 1381-94.

[12] National Coal Board. Subsidence engineering handbook. London: National coal board, Mining Department; 1975.

[13] Saeidi A, Deck O, Verdel T. Development of building vulnerability functions in subsidence regions from empirical methods. Engineering Structures. 2009;31:2275-86.

[14] Burland JB. Assessment of risk of damage to buildings due to tunnelling and excavation. Earthquake geotechnical engineering: Editions Ishihara, Balkema, 1995: 1189-201.

[15] Boone SJ. Ground-Movement-Related Building Damage. Journal of Geotechnical Engineering. 1996;122:886-96.

[16] Saeidi A. La vulnérabilité des ouvrages soumis aux aléas mouvements de terrains : développement d'un simulateur de dommages. PhD Thesis. Institut national polytechnique de Lorraine, Nancy, France; 2010.

[17] Saeidi A, Deck O, Verdel T. Development of building vulnerability functions in subsidence regions from analytical methods. Géotechnique. 2012; 62: 107-120.

[18] Wolfram S. Mathematica software version 6. USA: Wolfram Research, Inc.; 2008.

[19] Coulthard MA, Dutton AJ. Numerical modeling of subsidence induced by underground coal mining : Key questions in rock mechanics: Proc 29th US symposium, Minneapolis; 1988. P. 529–536.

[20] Ren G, Reddish DJ, Whittaker BN. Mining subsidence and displacement prediction using influence function methods. Mining Science and Technology. 1987;5: 89-104.

[21] Sheorey PR, Loui JP, Singh KB, Singh SK. Ground subsidence observations and a modified influence function method for complete subsidence prediction. International Journal of Rock Mechanics and Mining Sciences. 2000;37:801-18.

[22] Aissaoui K. Amélioration de la prévision des affaissements dans les mines à l'aide des approches empiriques, numériques et analytiques. PhD Thesis. Institut national polytechnique de Lorraine, Nancy, France; 1999.

[23] Saeidi A, Deck O, Al Heib Marwan, Verdel T, Rouleau Alain. Adjusting the Influence Function Method for Subsidence Prediction. Key Engineering Materials. 2013; 553: 59-66.

[24] Singh M. Mine subsidence. SME Mining Engineering Handbook, SME. 1992. pp. 938-971.

[25] Saeidi A, Deck O, Verdel T. Comparison of Building Damage Assessment Methods for Risk Analysis in Mining Subsidence Regions. Geotechnical and Geological Engineering. 2013;31:1073-88.

[26] Boscardin MD, Cording EJ. Building Response to Excavation-Induced Settlement. Journal of Geotechnical Engineering. 1989;115: 1-21.

[27] Boone SJ. Assessing construction and settlement-induced building damage: a return to fundamental principles. Underground Construction. Institu- tion of Mining and Metallurgy, London 2001; 559–70.

[28] Deck O, Singh A. Analytical model for the prediction of building deflections induced by ground movements. International Journal for Numerical and Analytical Methods in Geomechanics. 2012;36: 62-84.

[29] Finno RJ, Frank T. Voss J, Rossow E, Blackburn JT. Evaluating Damage Potential in Buildings Affected by Excavations. Journal of Geotechnical and Geoenvironmental Engineering. 2005;131: 1199-210.

[30] Selby AR. Tunneling in soils – ground movements, and damage to buildings in Workington, UK. Geotechnical and Geological Engineering. 1999;17: 351-71.

[31] Burd HJ, Houlsby GT, Augarde CE, Liu G. Modelling tunnelling-induced settlement of masonry buildings. Proceedings of the ICE - Geotechnical Engineering. 2000. p. 17-29.

[32] Melis M, Medina L, Rodríguez JM. Prediction and analysis of subsidence induced by shield tunneling in the Madrid Metro extension. Canadian Geotechnical Journal. 2002;39: 1273-87.

[33] Son M, Cording E. Estimation of Building Damage Due to Excavation-Induced Ground Movements. Journal of Geotechnical and Geoenvironmental Engineering. 2005;131: 162-77.

[34] Franzius JN PD, Burland JB. The response of surface structures to tunnel construction. Proceedings of ICE - Geotechnical Engineering. 2006;159: 3-17.

[35] Deck O, Anirudh H. Numerical study of the soil–structure interaction within mining subsidence areas. Computers and Geotechnics. 2010;37: 802-16.

[36] Spence RJS, Kelman I, Baxter PJ, Zuccaro G, Petrazzuoli S. Residential building and occupant vulnerability to tephra fall. Nat Hazards Earth Syst Sci. 2005;5: 477-94.

[37] Eguchi RT, Seligson HA. Loss Estimation Models and Metrics Risk Assessment, Modeling and Decision Support. Editors: Bostrom A, French S, Gottlieb S. Springer Berlin Heidelberg; 2008. p. 135-70.

[38] Reese S, Cousins WJ, Power WL, Palmer NG, Tejakusuma IG, Nugrahadi S. Tsunami vulnerability of buildings and people in South Java - field observations after the July 2006 Java tsunami. Nat Hazards Earth Syst Sci. 2007; 7:573-89.

[39] Al Heib M, Josien JP, El Shayeb Y. Paramètres d'affaissement pour la hiérarchisation des zones à risque dans le bassin ferrifère lorrain. Après –mines. Nancy 2003.

[40] Brady BHG, Brown ET. Rock mechanics for underground mining. Dordrecht, Netherlands: Kluwer Academic Publishers; 2004.

[41] Cauvin M, Verdel T, Salmon R. Modelling uncertainties in pillar stability analysis. International Journal on Risk Analysis, 2009; 29: 1371-1380.

Fig. 1. Description of the main characteristics involved in mining subsidence and their associated consequences]. a) Typical profiles of the ground displacements. b) Typical values of the subsidence dimension and ground movements. c) Typical damage due to mining subsidence in the city of Auboué, France.

Fig. 2. The main scheme of the damage simulator developed for the evaluation of the building vulnerability and hazard potential in a mining subsidence area.

Fig. 3. The principle of the influence function method. An infinitesimal mining element dA at depth H creates an elementary trough at the surface.

Fig. 4. Method used for the calculation of the horizontal displacement with IFM.

Fig. 5. a) The vertical section of the iron mining layers; b) the mining scenarios considered in the grey layer; c) the mining scenarios considered in the S2-S3 layer; d) the final five mine polygons considered for risk analysis of buildings in the town of Joeuf.

Fig. 6. Example of 10 hazardous polygons calculated between the minimal et maximal polygon 3 (scale factor of 0.5).

Fig. 7. A digitalized map of the town of Joeuf with the three building types.

Fig. 8. Limit vulnerability curves for the three building types, calculated with data of Table 4.

Fig. 9. City plot of the horizontal ground strain value and building damage for a single scenario (polygon 2).

Fig. 10. Histogram of the number of buildings affected by different classes of the horizontal ground strain and mean of damage μ_D for the five deterministic scenarios (polygon 1 to 5).

Fig. 11. City plots and histograms of the maximal damage (A) and the maximal horizontal ground strain value (B) for the 50 scenarios.

Fig. 12. City map and histograms of the damage mean value (A) and horizontal ground strain value (B) for the 50 scenarios under the equiprobable assumption.

Fig. 13. City plot and histograms of the damage mean value (A) and horizontal ground strain value (B) for the 50 scenarios under the non equiprobability assumption.

Fig. 14. Correlation between damage standard deviation and mean values

Fig. 15. Final ranking of buildings in the city, based on the maximal possible intensity and damage mean value, taking into account uncertainties.

Table 1. Characteristics of the mining layers and polygons under Joeuf [16].

Table 2. The values of influence angle depending on the nature of the ground at the boundary of each edge.

Table 3. Characteristics and relative probabilities for the five mining polygons.

Table 4. Parameters of the vulnerability functions for the three building types.

Table 5. Synthesis of the damage for the five deterministic scenarios (polygon 1 to 5).

Table 6. Scores used for each values of the maximal horizontal ground strain and the mean of damage for the final ranking.