Hadronic Mechanics Aspects of Irreversible Physical Pendula

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Abstract. All macroscopic physical pendula undergo various types of damping processes which make them irreversible devices. Their repeated use for detecting cosmological micro-anomalies requires the determination of certain observables with very high precision despite an increased variance due to the strictly nonperiodical pendulum behaviour. Using genofields from Hadronic Mechanics, data processing algorithms involving backward and forward convolutions have been developed. They greatly improve precision in the determination of swinging azimuth, swinging amplitude, swinging period and precession period. To the author's knowledge, the very weak anisotropy of a long Foucault pendulum has been characterized for the first time experimentally in terms of zero-amplitude swinging period and conservative (Hamiltonian) amplitude oscillations.

Keywords: Physical pendulum, Foucault pendulum, irreversibility, hadronic mechanics.

PACS: 02.20.Sv Lie Algebras of Lie groups
05.30.Bp Spatial dimensions
07.05.Kf Data analysis: algorithms and implementation; data management

INTRODUCTION

The simple pendulum is idealized in practically all textbooks as a harmonic oscillator obeying Hamilton's mechanics. However, the mere presence of some inevitable damping in a physical one-degree-of-freedom pendulum prevents the trajectory in phase space from being closed and periodical. To study the damping phenomenon, the phase space trajectory is made artificially closed by making the energy loss over one half-cycle stored in some imaginary potential reservoir, and then given back to the pendulum in a time reversal process for the next half-cycle, thus preserving the Hamiltonian description. [1] The Foucault pendulum has been thoroughly studied in his 1879 dissertation as a two-degrees-of-freedom harmonic oscillator by Kamerlingh Onnes [2], who was the first to apply the perturbation methods of celestial mechanics to the Foucault pendulum. He considered viscous damping at very small amplitudes using a perturbation Hamiltonian. However every physical pendulum with swinging amplitudes over a few centimetres undergoes dominant aerodynamic damping with different power loss parameters for each degree of freedom. Moreover, physical pendula show a non negligible 3rd degree of freedom [3] in the form of torsion about the suspension wire or rod axis (a sort of macroscopic spin degree of freedom) again with its own damping parameters. Nonlinear coupling cause conservative energy transfer between the three degrees of freedom.

It is customary that pendulum experimenters try to measure observables by averaging their values over tens or hundreds of cycles in order to increase accuracy. However, that process has its limits since the variance increases with time due to irreversible processes, so that the instant of the measurement itself becomes inaccurate. In the following, typical difficulties encountered in measuring pendulum properties are presented. It will be shown how defining appropriate genounits can lead to pendulum equations that are spin invariant and time invariant over a larger scale, thus allowing accurate determination of observables such as the zero-amplitude period of oscillation, the azimuths of oscillation and ellipticities at prescribed times, etc. The hadronic mechanics treatment of physical pendula not only accounts for the non-Hamiltonian irreversibility of the damping but also improves the accuracy on observables.

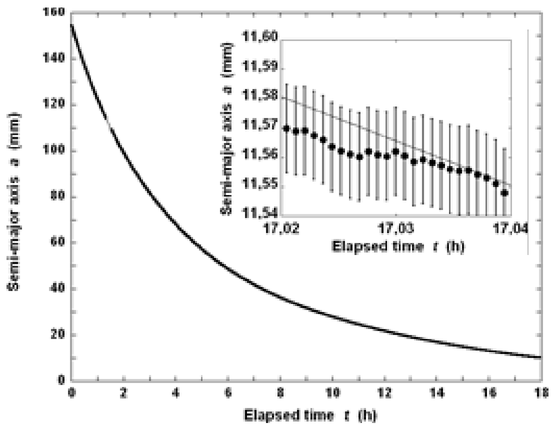
IRREVERSIBLE SWINGING AMPLITUDE

Figure 1 illustrates the most drastic irreversible property of a Foucault pendulum: the damping responsible for the monotonous decrease of its amplitude. Applying perturbation methods to the linearly damped harmonic oscillator as an exact zero-order solution [4], the first order perturbation treatment of aerodynamic drag yields a virtually exact formula for the irreversible attenuation of the amplitude a along the semi-major axis of a pendulum with swinging period T [5]:

$$a = a_0 \big/ (e^{\alpha t} + \beta t) \tag{1}$$

where $a_0 = 154.356$ mm is the amplitude at $t = 0$; $1/\alpha = 6.6517$ h is the viscous time constant; $1/\beta = 3T \big/ 8\delta a_0 = 10.065$ h is some sort of aerodynamic time constant; $\delta = 0.000387\ \mathrm{m}^{-1}$ is taken from a "mild" version of Santilli's Equation (5.17) in Ref. [6]:

$$m\, dv \big/ dt = k_1 v^1 + k_2 v^2 \qquad \text{with} \quad \alpha = k_1/2; \delta = k_2 \ . \tag{2}$$



(a)

$a = a_0/(\exp(\alpha t)+\beta t) + a_1\cos(\phi_1+2\pi t/20.68) + a_2\cos(\phi_2+2\pi t*2/20.68) + a_3\cos(\phi_3+2\pi t*3/20.68)$		
	Value	Error
a_0 (mm)	154.356	0.003
$1/\alpha$ (h)	6.6517	0.0006
$1/\beta$ (h)	10.065	0.004
a_1 (mm)	1.307	0.002
ϕ_1 (rad)	1.392	0.002
a_2 (mm)	0.3049	0.0006
ϕ_2 (rad)	6.011	0.002
a_3 (mm)	0.0386	0.0003
ϕ_3 (rad)	4.809	0.006
Residuals (mm)	0.015	

(b)

FIGURE 1. (a) Swinging amplitude of an 8-meter Foucault pendulum that ran for 18 hours in Gifu, Japan, on July 21, 2009. The amplitude decay is perfectly described by Equation 1 within 15 µm rms. In the inset, a detail of the fit of the amplitude decay law to a small domain of the 22500 experimental points measured at every half swing, thanks to an algorithm involving a genounit. From the fitted equation in (b), it can be seen that, once the irreversible phenomenon has been accounted for, extremely faint observables such as the Hamiltonian exchanges between pendulum modes can be measured up to the 3rd harmonic.

Since the above pendulum is meant to detect possible micro-anomalies of the gravitational field, the necessary extreme precision is achieved by using non intrusive videographic remote sensing of a set of luminous marks on the pendulum bob and on its reference alidade. Significant bob positions are established 170 times during a swing cycle, enabling the determination, at every half-cycle, of the semi-axes a and b of an "observable" called "ellipse". However, at the precision needed, the bob orbit in the laboratory frame is rather the superposition of an ellipse and a hypocycloid, both of them being modulated by a decreasing spiral. In the data processing algorithm designed to study Fig. 1, an invariant ellipse arc is reconstructed for every sequence of 42 images centered on the instant of passage across a semi-axis extremity. Within the convolution process, each image is genomultiplied by an appropriate genounit to compensate back and forth for the non-Hamiltonian damping and for the Hamiltonian precession due to earth rotation. Then the ellipse parameters become true observables.

IRREVERSIBLE SWINGING PERIOD

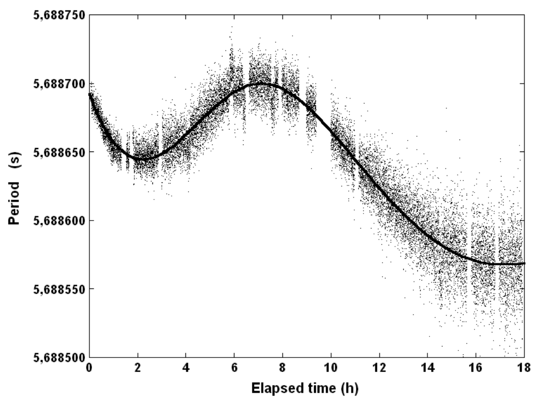
In a similar manner, to access the observable "zero-amplitude swinging period" T_0, an algorithm must cope with the irreversible decrease of the period with time due to nonlinearity [2], following the decrease in amplitude:

$$T(t) = T_0\sqrt{1+\frac{a^2(t)}{8l^2}} \approx T_0\left(1+\frac{a^2(t)}{16l^2}+...\right) \qquad (a \ll l) \tag{3}$$

where l is the pendulum length .

By differentiating Equation 1, the reader may verify that, for the first few hours when $t < \alpha^{-1} < \beta^{-1}$,

$$T(t) \approx T_0\left(1+(T(0)-T_0)e^{-2(\alpha+\beta)t}\right) \tag{4}$$



(a)

$T = T_0 + T_\gamma \exp(-t/\gamma)$
$+ T_1 \cos(\phi_1 + 2\pi\, t/20.68)$
$+ T_2 \cos(\phi_2 + 2\pi\, t*2/20.68)$
$+ T_3 \cos(\phi_3 + 2\pi\, t*3/20.68)$

	Value	Error
T_0 : mean period (s)	5.6886214	0.0000014
T_γ : transient amplitude (s)	0.0001118	0.0000033
γ : transient time constant (h)	2.02	0.16
T_1 : fundamental amplitude (s)	0.0000634	0.0000003
ϕ_1 : fundamental phase (rad)	-2.25	0.03
T_2 : 2nd harmonic amplitude (s)	0.0000113	0.0000008
ϕ_2 : 2nd harmonic phase (rad)	1.80	0.04
T_3 : 3rd harmonic amplitude (s)	0.0000020	0.0000002
ϕ_3 : 3rd harmonic phase (rad)	0.71	0.18
Rms residual (s)	0,000013	

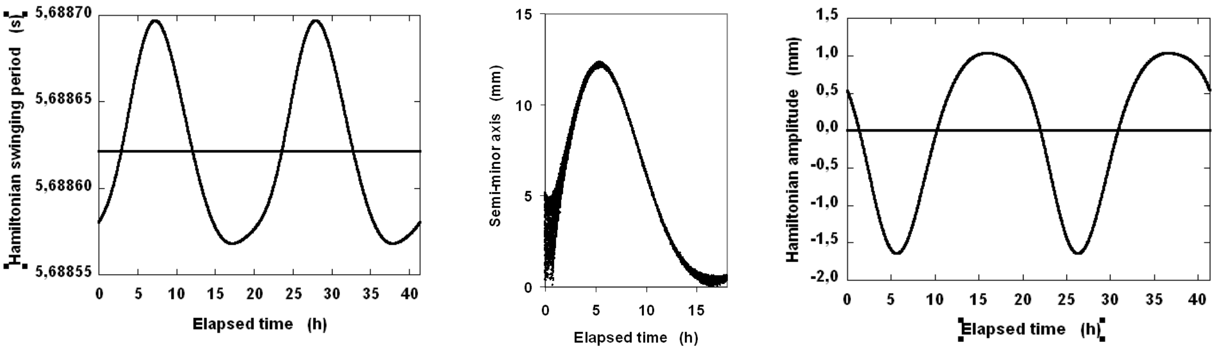
(b)

FIGURE 2. Evolution of the period of oscillation of an 8-meter Foucault pendulum during a continuous 18-hour experiment in Gifu, Japan, on July 21, 2009. Except for a few missing data, some 22 500 individual half-cycle period measurements are fitted by the curve. A periodic variation (Hamiltonian) at half the Foucault precession period (20.68 h) is characteristic of an anisotropy of the field in which the pendulum oscillates. However, due to an irreversible, non Hamiltonian, transient phenomenon during the first hours, the invariant observable "zero amplitude period" is not readily accessible for measurement.

Figure 2 very neatly shows the transient change in the measured period during the first hours of an experiment. The transient time constant from the fitted curve is indeed (2.02 ± 0.16) h for the Gifu pendulum, in perfect agreement with the value of (2.005 ± 0.001) h obtained from Fig. 1b and Equation (4). Most experimenters use the pendulum for a duration of the order of 1 hour. Therefore, their period measurements cannot be considered independent of amplitude damping and, as such, are negatively affected by irreversibility.

ANISOTROPY CHARACTERISTICS AS TRUE OBSERVABLES

It has been shown by Kamerlingh Onnes [3] that suspension anisotropy is responsible for two non degenerate eigenmodes with different swinging periods at 90° azimuths from one another. Starting the pendulum in a quadrant between those eigenazimuths gives rise to elliptical orbits whose major axis will show alternating precession velocities with a 180° cycle, independently of the 360° Foucault precession cycle. As the pendulum sweeps trough the azimuths due to Foucault precession, each eigenazimuth is encountered once every 180°, but not symmetrically in time because of the fluctuating precession speed due to elliptical orbits (hence there is generation of harmonics). In Fig 2b, fitting a three-harmonic wave to the data yields a long-term mean swinging period of (5.688621 ± 0.000001) s and fundamental wave period of 20.72 h, in excellent agreement with the half-Foucault precession period in Gifu.



(a) (b) (c)

FIGURE 3. (a) Evolution of the zero-amplitude Hamiltonian swinging period, unaffected by the damping process, for a complete Foucault precession period of 41.36 h. (b) Evolution of the ellipse semi-minor axis as a consequence of the azimuth dependence of the period due to suspension anisotropy. (c) Evolution of the Hamiltonian, energy conserving amplitude of the semi-major axis, unaffected by the damping process, due to the energy taken by the semi-minor axis.

It is interesting to note in Fig. 3c that the amplitude wave with a fundamental period at half the Foucault precession period in Gifu (20.68 h) is not affected by the damping process. It shows precisely the negative waveform of the semi-minor axis (Fig. 3b) as an energy conservation exchange between the two vibrating modes.

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