DEVELOPMENT OF BUILDING VULNERABILITY FUNCTIONS IN SUBSIDENCE REGIONS FROM EMPIRICAL METHODS

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ABSTRACT

The extraction of ore and minerals by underground mining often causes ground subsidence phenomena. In urban regions, these phenomena may induce small to severe damage to buildings. To evaluate this damage, several empirical and analytical methods have been developed in different countries. However, these methods are difficult to use and compare due to differences in the number of criteria used (from 1 to 12). Furthermore, the results provided by damage evaluation may be significantly different from one method to another. The present paper develops vulnerability functions based on a concept that has been applied in other areas, such as earthquake engineering, and that appears to be a more efficient way to assess building vulnerability in undermined cities. A methodology is described for calculating vulnerability functions in subsidence zones using empirical methods. The first part of the paper focuses on existing empirical methods for damage evaluation, and selected necessary improvements or modifications are justified. The second part focuses on the development of a building typology in subsidence zones and its application in the Lorraine region, where many villages are subject to subsidence problems due to iron-ore mining. The third section describes and discusses the adopted methodology for determining vulnerability and fragility functions or curves. Finally, vulnerability functions are tested and validated with a set of three subsidences that occurred in Lorraine between 1996 and 1999.

KEYWORDS: Vulnerability, mining subsidence, damage, horizontal ground strain, fragility curve, vulnerability curve.

1. Introduction

1.1. Context and objectives

The extraction of ore and minerals by underground mining may induce ground subsidence phenomena. These phenomena lead to horizontal and vertical ground movements, and consequently to deformation and damage to buildings in urban undermined regions. The maximum vertical displacement occurs in the center of the subsidence area and may reach several meters. This displacement is accompanied by horizontal strain in the ground, ground curvature, and slopes, which make up the three types of movements that load structures and cause structural damage [1]. Therefore, subsidence may induce small to severe damage in buildings (see Fig. 1). Many countries have concerns about abandoned mines and mitigation of risk due to subsidence hazard is therefore an important issue. The Lorraine province in French is a typical region with more than 140 km^2 of abandoned underground mines.

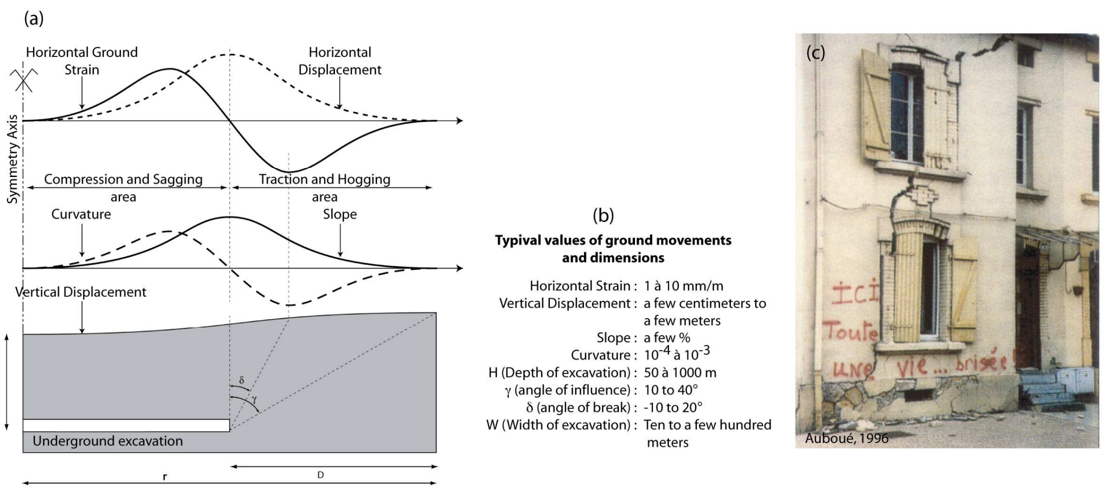

Fig. 1. Description of the main characteristics involved in mining subsidence a) description of ground movements, b) typical values of ground movements and dimensions, c) typical building damage.

Building damage assessment in mining subsidence hazard areas is a key point for risk management. The main obstacle is that existing methods, developed in different countries, are more appropriate for the study of single buildings than for large urban areas. Most of these are empirical and based on databases of observed building damage in subsidence regions of these countries (see Appendix). Application of these methods raises the following questions: Which method is the most efficient and accurate? Which method is the most conservative? A basic comparison [2] shows that the application of these methods at the same site does not give exactly the same results because each method was developed in a specific context (geology, mining characteristics, and so on) and because each method uses a different set of parameters, two parameters for the simplest and thirteen for the most detailed (see Appendix). Methods with a lesser number of parameters may appear most efficient but are probably less accurate. In contrast, the most complex methods may appear more accurate but their application in cities with several hundreds of buildings is a long process. Moreover, it may be assumed that all of these methods may be consistent but do not focus on the same parameters. Consequently, in the case where no justification exists to choose or favor one method, the mean value of the damage calculated with each of the methods may appear to be the most relevant result but also less efficient.

What is the uncertainty in the damage assessment? All of these methods make a deterministic evaluation of the damage, but experience shows that similar adjacent buildings affected by the same subsidence may suffer different damage. The problem of the uncertainties for building damage assessment is addressed in other fields of risk analysis, such as seismic engineering [3], [4], volcanic engineering [5], and tsunamis [6]. It is based on the use of vulnerability and fragility curves to assess the mean amount of damage and the damage distribution of all buildings with similar characteristics in relation to the event intensity. This approach has proven to be a good compromise between accuracy of the results and necessary investment for the studies (cost and duration). The development of such a method in the field of mining subsidence would then be innovative and efficient for use in assessment of the possible damage that an entire city may suffer from mining subsidence.

The objective of this study is to develop vulnerability curves and functions, starting from empirical methods used for individual buildings, for a method able to deal with a large quantity of buildings.

Below, we first present the concepts of vulnerability and fragility curves. In Chapter 2, the development methodology for these curves in mining subsidence hazard areas is described. Next, in Chapter 3, we discuss the existing empirical methods for damage evaluation and justify some essential modifications and harmonizations. Chapter 4 describes the typology used to develop the fragility and vulnerability curves. Then, the methodology used for development of the vulnerability and fragility curves is presented and discussed (Chapter 5). Finally, as an application and validation, results are compared to the observed damage in three villages of the French Lorraine region (France).

1.2. *Vulnerability and fragility curves concepts*

The vulnerability of buildings and territories to natural hazards is often studied with vulnerability and fragility curves that allow assessment of the damage distribution for a given number of building types in relation to the event intensity (e.g., earthquakes [3] [4], volcanoes [5], and tsunamis [6]). Fragility and vulnerability curves are thus developed for a given building type, and allow quick and realistic damage assessment of all buildings grouped into the same type.

Vulnerability and fragility curves use the following three main types of input data:

1. A damage scale
2. A building typology
3. An intensity criterion

For example, the EMS-98 [7] considers a six-level damage scale that consists of: no damage (D0), slight damage (D1), and so on, up to very heavy damage (D5). Most of the existing methods define an equivalent number (four levels of damage in the HAZUS [4] and six levels in volcanic risk assessment [5]). Four levels are considered in this study (see Chapter 3).

Building typology must be defined according to the most important parameters relevant to resistance of the buildings against the considered hazard. For instance, the building materials (concrete, wood, masonry, etc.), the quality of construction, the type of foundations, and the global stiffness of the building are important in earthquake engineering [7]. The EMS-98 considers 11 main building types [7]. Five building types are investigated in this study (see Chapter 4).

The criteria for the event intensity may be a physical parameter (height or speed for a tsunami, acceleration for an earthquake) or an empirical one (earthquake intensity in EMS-98 [7]). In this study, the horizontal ground strain parameter is considered as the criterion for event intensity (see Chapter 4).

Fragility curves provide the probability of reaching or exceeding a given damage state as a function of the intensity of the natural event (Fig. 2b), and they are usually modeled by lognormal functions. A very important point is that fragility curves clearly take into account that not all buildings of the same type will suffer the same level of damage for a given event intensity.

Vulnerability curves are relationships between the mean amount of damage for a given type of building and the value of the event intensity (Fig. 2c). Vulnerability curves may be deduced from fragility curves with Eq. (1):

$$\mu_D = \sum P_k \cdot D_k \qquad (1)$$

where μ_D is the mean damage for a given intensity, P_k is the probability of a damage grade D_k, and k is the range of damage category (from 0 to 5 in the EMS-98 damage scale, for instance [8]).

An example of these curves is shown in Fig. 2 for a massive stone masonry building (type M4), according to the EMS-98 [7]. Fig. 2a shows the damage distribution for this type of building during an earthquake of intensity 11. This distribution can be plotted in Fig. 2b, where each dot on the figure corresponds to the different fragility curves of this type of building. By calculating the mean of the damages (Eq. 1), it is then possible to plot one point of the vulnerability curve, as shown in Fig. 2c. Fragility and vulnerability curves may be then modeled by fitted mathematical functions.

In practical terms, when developed and validated, fragility and vulnerability curves are both efficient and accurate. Vulnerability curves are used to obtain a synthetic result of the mean damage to buildings in a selected territory. When applied to a single building, fragility curves may be used to assess the probability of reaching a particular damage level. When applied to a set of buildings, fragility curves may be used to assess the damage distribution of all buildings.

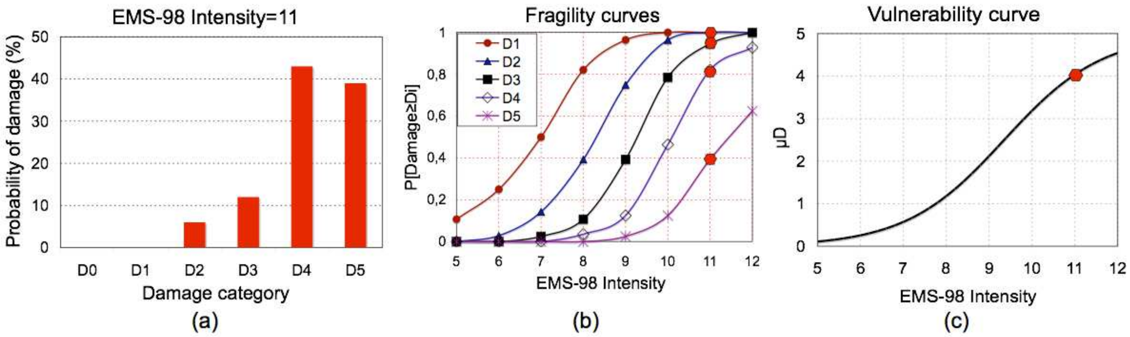

Fig. 2. Damage distribution (a), fragility curves (b), and vulnerability curves (c), for the M_4 building type, according to EMS-98 [7] for assessment of earthquake building damage.

2. Development methodology of vulnerability and fragility curves for buildings in a subsidence zone

The methodology adopted in this study to develop vulnerability and fragility curves is shown in Fig. 3. After selecting the three main input data, a damage evaluation method must be chosen from among the different building damage assessment methods for subsidence engineering (five empirical methods have been considered in this study). A representative sample of each building type is then constructed with 1000 simulated buildings. Then, for the different possible values of the horizontal ground strain (around twenty possible values between 0 and 10 mm/m) the damage to all sample buildings is assessed. In the next step, both the mean damage for a given value of the intensity and the probability of reaching or exceeding a given damage state can be calculated. Finally, by repeating the previous step for all values of the horizontal ground strain, both the vulnerability and fragility curves can be drawn.

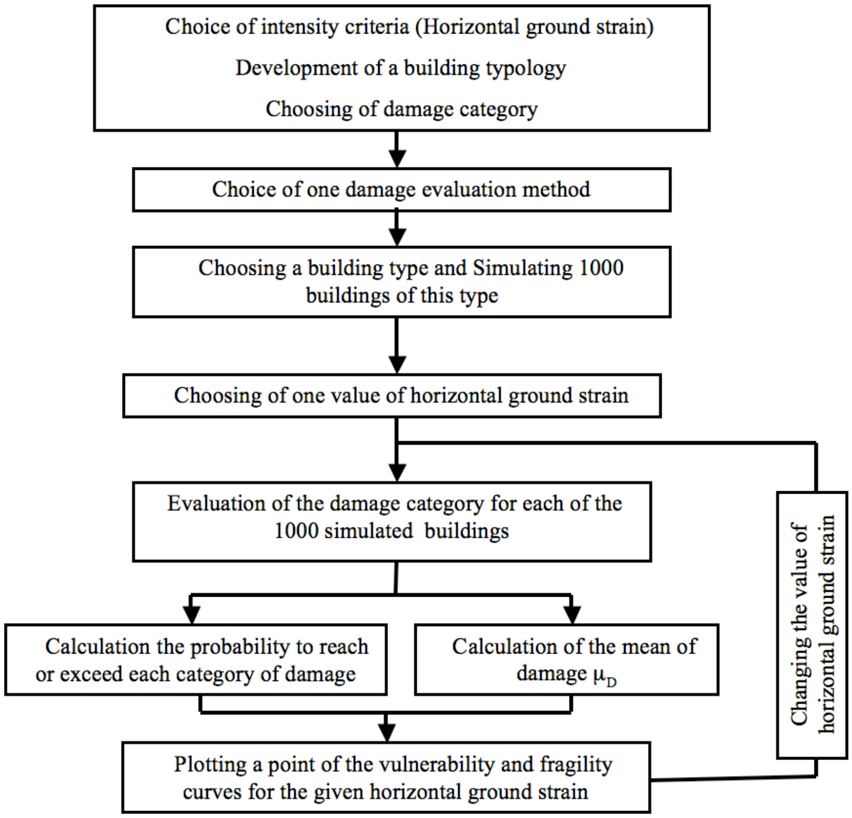

Fig. 3. Methodology for determination of vulnerability and fragility curves in a subsidence zone.

3. Choice of damage evaluation method

Existing methods for building damage assessment were developed in different countries where mining subsidence has occurred (e.g., England, USA, Poland, South Africa). Most of them are empirical and use observational data from damaged buildings. Only a few of these methods use mechanical analysis. These methods are described in the Appendix and may be classified as either abacus or rating methods [1]. Table 1 summarizes some characteristics of these methods.

Table 1. Summary of building damage assessment methods used in mining subsidence hazard areas.

Method	*Type*	*Empirical/ Analytical*	*Damage scale*	*Definition of vulnerability classes*
NCB [9]	Abacus	Empirical	5 levels (Fig. 4-a)	No
Wagner and Schumann[10]	Abacus	Empirical	5 levels (Fig. 4-b)	No
Boscardin and Cording [11] or Burland[12]	Abacus	Analytical	5 levels (Fig. 4-c)	No

Bhattacharya and Singh [13]	Rating	Empirical	3 levels (Table 9)	yes (4 classes) (Table 9)
Yu *et al.* [14]	Rating	Empirical	3 levels (Table 8)	yes (4 classes) (Table 8)
Dzegeniuk *et al.* [15]	Rating	Empirical	5 levels (Table 10)	yes (5 classes) (Table 10)
Kwiatek [16]	Rating	Empirical	No damage scale	yes (5 classes) (Table 11)

The abacus methods, namely the National Coal board (NCB) method [9] and the Wagner and Schumann method [10] (Fig. 4a and 4b), link building damage to building length and the horizontal ground strain. These two methods are very similar, although they were developed from a database of observations resulting from coal mining in the UK and in South Africa, respectively. Detailed comparisons [2] lead to the conclusion that the two methods may be considered as the same. The methods of Boscardin and Cording [11] or Burland [12] are based on a simple mechanical model analysis and use two parameters: the angle of distortion (β) or deflection (Δ), which depend on the length of the building (L), the ground curvature (1/R) ($\beta = L/2R$, $\Delta = L2/8R$), and the horizontal ground strain. These methods both use the same number of damage levels but with different interpretations (Fig. 4c). In the following, we will not refer to these analytical methods because they were developed for one dimension of the buildings (height equal to the length) and for unreinforced masonry buildings only. Their use would require adaptation to the final abacus for particular dimensions and mechanical properties. This would necessitate specific developments that are not involved in the scope of this study.

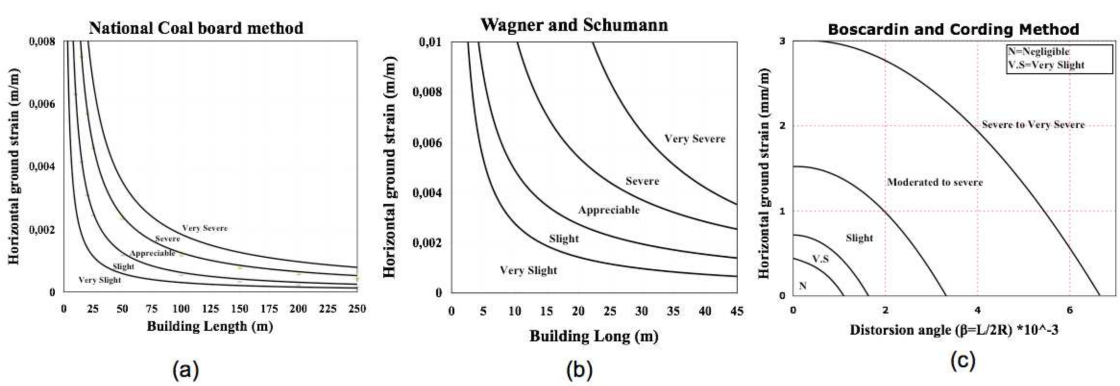

Fig. 4. Schemes of NCB [9], Wagner and Schumann [10] and "Boscardin and Cording" methods [11] for the assessment of building damage in mining subsidence hazard areas.

The rating methods consider several parameters, such as building length, building shape, building typology, and type of foundation, to define the vulnerability class of a given building. A set of threshold values of horizontal ground strain is then used to define the damage level in relation to the horizontal ground strain. The main differences are the criteria used to define the vulnerability class, the number of vulnerability classes (between 4 and 5) and the number of damage classes (shown in Table 1 and Table 2). One is an American method developed by Yu *et al.* [14, Table 8 in Appendix], another is the Bhattacharya and Singh method [13, Table 9 in Appendix], and two others are methods developed by Dzegniuk *et al.* [15, Table 10 in Appendix] and by Kwiatek [16, Table 11 in Appendix]. The Yu *et al.* method [14] appears to be a more comprehensive version of the Bhattacharya and Singh method. Therefore, the Bhattacharya and Singh method is not used to develop vulnerability and fragility curves.

In order to develop fragility and vulnerability curves with all of the empirical methods, some slight modifications and harmonizations are necessary so that all methods use the same number of damage levels and all rating methods use the same number of vulnerability classes. Details of this work are justified in the Appendix. Finally, four damage classes are used for all methods (Table 2). For the rating methods, four building vulnerability classes and sixteen threshold values of the horizontal ground strain are defined (Table 3).

Table 2. Comparison between the four damage classes used and the damage classes defined in existing methods.

<table>
<tr><th>Unified Damage Class used</th><th>Yu et al. [14] or Bhattacharya and Singh [13]</th><th>Pellisier et al. [17]</th><th>NCB[9]</th><th>Burland [12] or Boscardin and Cording[11]</th><th>Bruhn et al. [18]</th><th>Ji-Xian[19]</th><th>Kwiatek [16] or Dzegniuk et al. [15]</th></tr>
<tr><td rowspan="2">D_1</td><td rowspan="3">Architectural</td><td>No damage</td><td rowspan="2">Very Slight</td><td>Negligible</td><td rowspan="3">Slight</td><td rowspan="2">1</td><td rowspan="6">No Damage Class</td></tr>
<tr><td>Very Slight</td><td>Very Slight</td></tr>
<tr><td>D_2</td><td>Slight</td><td>Slight</td><td>Slight</td><td>2</td></tr>
<tr><td>D_3</td><td>Functional</td><td>Moderate</td><td>Appreciable</td><td>Moderate</td><td>Moderate</td><td>3</td></tr>
<tr><td rowspan="2">D_4</td><td rowspan="2">Structural</td><td>Severe</td><td>Severe</td><td rowspan="2">Severe
Very Severe</td><td>Severe</td><td rowspan="2">4</td></tr>
<tr><td>Very Severe</td><td>Very Severe</td><td>Very Severe</td></tr>
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Table 3. Threshold values of the horizontal ground strain used to assess building damage with the rating methods in relation to the vulnerability class of a given building.

Vulnerability Class / Damage Class	C_1	C_2	C_3	C_4
D_1(negligible or very slight)	<0.5 mm/m	<1.5mm/m	<2.5 mm/m	<3.5 mm/m

D_2(Slight)	0.5-1.5 mm/m	1.5-2.5 mm/m	2.5-4.5 mm/m	3.5-6 mm/m
D_3(Moderate)	1.5-2.5 mm/m	2.5-3.5 mm/m	4.5-6 mm/m	6-9 mm/m
D_4(Severe and very severe)	≥2.5mm/m	≥3.5 mm/m	≥6 mm/m	≥9 mm/m

Finally, four empirical methods are available for developing vulnerability and fragility curves in mining subsidence hazard areas: one abacus (the NCB [9] or Wagner and Schumann [10]) and three rating methods (Yu *et al.* [14], Dzegniuk *et al.,* [15] Kwiatek [16]). The development of the vulnerability and fragility curves may then be based on one of these methods if considerations regarding the mining and building context justify that one method will be more relevant and accurate. In the case where no justification is possible, the mean value of the damage "MD" given by all the methods is assumed to be the most probable damage assessment (Eq. 2).

$$MD(\varepsilon) = \frac{NCBWD(\varepsilon) + YD(\varepsilon) + DZD(\varepsilon) + KD(\varepsilon)}{4} \quad (2)$$

MD(ε) is mean damage of the four empirical methods, and NCBWD(ε), YD(ε), DZD(ε), and KD(ε) are the damage levels for a building with a horizontal ground strain of "ε" assessed with the empirical methods (respectively NCB[9] or Wagner and Schumann [10], Yu *et al.* [14], Dzegniuk *et al.* [15], and Kwiatek [16]).

4. Development of building typology in mining subsidence areas

4.1. Relevant parameters of buildings for a typology

The identification of relevant parameters is based on the following points:

Relevant parameters are mainly chosen from the list of criteria used in the empirical methods and on the accepted perception of the loading process of the buildings [20]. The building length is one of the most important parameters. The building resistance is also an important parameter because both the horizontal ground strain and the associated ground curvature lead to an increase of stresses in the structure (compressive and tensile). Finally, the connection with the ground appears to be fundamental; deeper foundations provide a greater underground surface of the building on which the ground may thrust. Foundations of buildings and their resistance against a lateral load must then be investigated.

The bibliography concerning building vulnerability in the fields of earthquake engineering (EMS-98 [7] and HAZUS [4]) and volcanic engineering [5] shows that a typology must not be too complex to be operational, and some building particularities may still be taken into account in a second step. These typologies also emphasize the importance of exterior and interior symmetry of the load-bearing walls of the building in a more detailed way than that used in the Kwiatek[16] and Dzegniuk *et al.* [15] methods.

In light of this, four main parameters are selected for the typology: structural material, length, foundations, and shape. Each parameter may thus include several criteria from the existing empirical methods. For instance, the protection system used in the Kwiatek [16] and Dzegniuk *et al*. [15] methods is considered in the building materials and foundations system criteria. The following sections will explain each of these four parameters.

Structural materials

We selected four types of structural materials with up to three subcategories. We mainly detailed structural materials for masonry and reinforced concrete structures because most of the buildings in the Lorraine region in France are of those types. Several categories are similar to those defined in the EMS-98 [7], such as rubble stone / fieldstone, unreinforced brick / concrete blocks, reinforced brick, and confined masonry.

a) Masonry structure:

- Poor masonry that consists of rubble stones, fieldstones, and adobe or earth bricks with poor quality mortar without protection against mining subsidence effects (MR).
- Good masonry that consists of bricks or concrete blocks with good quality mortar and with a possible weak reinforcement (MB).
- Reinforced and confined masonry that consists of bricks or concrete blocks with good quality mortar and with horizontal and vertical reinforcement (MC).

b) Reinforced concrete structure:

- Reinforced concrete frame structure (CF)
- Reinforced concrete shear wall structure (CS)

c) Steel Structure (ST)

d) Wooden Structure (WO)

4.1.1 Building length

According to the threshold values of length used in the empirical methods, and also according to the traditional length of buildings in the Lorraine region, five categories of building length have been defined (Table 4).

Table 4. Classification of building lengths.

Description	Length value	Group name
Low	Less than 10 m	L
Medium	Between 11 and 20 m	M
High	Between 21 and 30 m	H
Very high	Between 31 and 40 m	V
Exceptional	More than 41 m	E

4.1.2 Building foundations and basement

Building foundations have been classified into nine categories depending on their depth into the ground and their resistance against lateral load (Fig. 5).

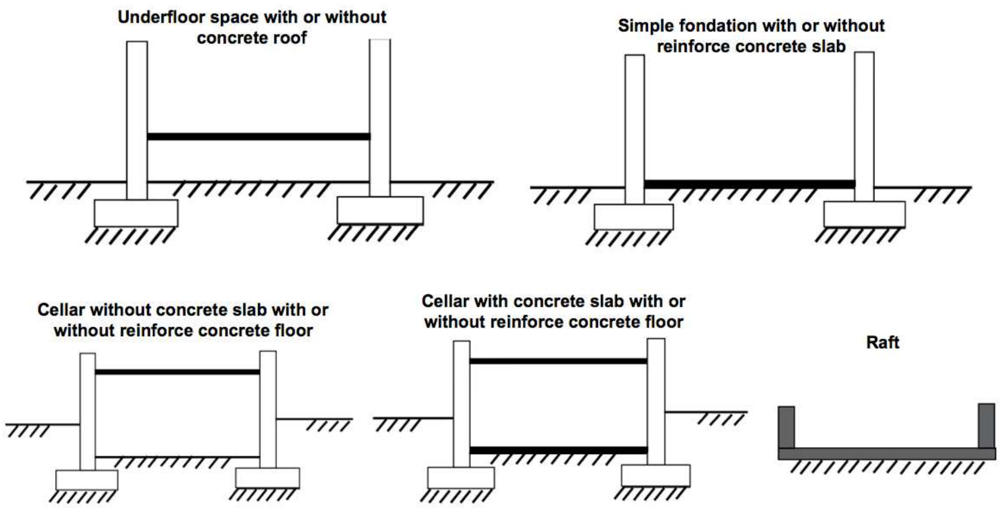

Fig. 5. Types of foundations considered in the typology.

- Foundation under floor space with reinforced concrete floor (VB) or without concrete floor (VS)
- Simple foundation with or without reinforced concrete slab (SB, SS)
- Cellar without concrete slab and with or without reinforced concrete floor (CB, CC)
- Cellar with concrete slab and with or without reinforced concrete floor (DB, DS)
- Raft foundation (RE)

4.1.3 Building shape and symmetry

Six categories have been defined depending on the simplicity or compactness of the external shape, the regularity of the external shape, and the symmetry of the interior bearing walls.

- Simple external shape with good symmetry or bad symmetry of the bearing walls (SR, SN).
- Little dismembered external shape with good symmetry or bad symmetry of the bearing walls (LR, LN).
- Strongly dismembered external shape with good symmetry or bad symmetry of the bearing walls (FR, FN).

4.2. *Application of the typology for buildings in the ferriferous basin in France*

The described typology leads to 1890 theoretical building types (7 “materials” x 5 “lengths” x 9 “foundations” x 6 “shapes”). In the ferriferous basin region, most of the buildings are worker housing estates with similar characteristics and are constructed of masonry [21]. Around 70% of the buildings in these regions may be grouped into five types (Table 5). The name of each type is constructed by merging the name of each parameter. For instance, MRHDBFN is a building type that encompasses “unreinforced masonry buildings (MR)” with “high length (H)”, a “cellar with concrete slab and reinforced concrete floor (DB),” and a “strongly dismembered shape with bad symmetry of the internal bearing walls (FN)”. A simplified name is given in the first column of the Table 5.

Table 5. Building typology in Lorraine region.

Type’s Name	Material	Length	Foundation	Form	Typology Name
MR1	(MR)	(M)	(SS)	(SR)	MRMSSSR
MR2	(MR)	(M)	(DB)	(SR)	MRMDBSR
MR3	(MR)	(H)	(DB)	(LR)	MRHDBFN
MC2	(MC)	(M)	(DB)	(LR)	MCMDBLR
CF1	(CF)	(M)	(DB)	(SN)	CFMDBSN

5. **Development of vulnerability and fragility curves**

5.1. *Development of a simulated building database*

The methodology used to develop vulnerability and fragility curves is described in Fig. 3. The horizontal ground strain is used as intensity criteria, a building typology is developed, and five methods are made available to assign the building damage into one of four categories. The details of the methodology are illustrated with the CF1 building type.

The next step of the methodology is to develop a representative database of the building types with 1000 theoretical buildings. Preliminary tests, with a number of buildings between 200 and 2000, showed that 1000 buildings provided acceptably accurate results.

To complete this database, the variability of each criteria used in the different methods of damage assessment is considered to be in agreement with the building type (Table 6). A uniform statistical distribution is used to define the final value of each building. This variability within a building type may be interpreted both as a real physical and observed difference between the buildings and also as uncertainties concerning their real characteristics.

For example, the length parameter for this type is classified into the medium range, and this parameter varies from 10 to 20 m. The lengths of the 1000 buildings are then randomly chosen between 10 m and 20 m. The other parameters of the typology (material, foundation, and shape) also lead to some uncertainties according to the empirical methods used for the analysis. Therefore, the development of vulnerability functions requires that we properly define the variability of each parameter used by the method into each building type for each empirical method. The resulting variability in the CF1 building type is described in Table 6.

Table 6. Variability of the criteria used by each empirical method in the "CF1" building type for the simulation of the buildings. Values of the building length, building height, and subsidence radius are metric values. Other values in the table are rates that are defined for each original method (see Appendix).

		Building													Ground and subsidence	
Building type	Empirical method	Length [m]	Height [m]	Material	Rigidity	Foundation	Material of foundation	Shape	Cellar wall	Floor storeys	Lintel	Protection	Condition	Pre damage state	Ground type	Radius of subsidence [m]
CF1	NCB or Wagner and Schumann [9]	10-20	-	-	-	-	-	-	-	-	-	-	-	-	-	-
	Yu *et al.* [14]	10-20	5-8	2-4	-	4	-	-	-	-	-	-	-	-	-	100-500
	Dzegniuk *et al.* [15]	10-20	-	-	0-2	0-1	-	1-5	-	-	-	0-1	0-4	-	8-12	-
	Kwiatek [16]	10-20	-	-	-	0-2	0-2	2-4	0-1	0-2	1-2	0-2	0-2	0-2	4-8	-

All parameters related to the building or foundation material, type of foundation, building shape, type of cellar wall, and type of floor story are determined directly according to the name of each building type.

The rigidity and protection parameters depend on the two categories of "structure material" and "structure foundation". The technical condition and pre-damage state parameters used in the Kwiatek [16] method are determined by factoring the "structure material" category into the typology. In Lorraine, unreinforced masonry buildings make up the oldest buildings, and technical conditions for these are generally worse than for the reinforced concrete buildings.

Some parameters, such as the building height, the subsidence radius parameter (Yu *et al.* method [14]), different elements of construction (e.g., type of lintel in the Kwiatek [16] method), and the ground type (Dzegniuk *et al*. [15] and Kwiatek [16] method), cannot be defined directly by the building typology and should be adapted in each particular case. We chose the more realistic and probable values for the typical buildings of the Lorraine region: radius of the subsidence between 100 m and 500 m, height of the buildings between 5 m and 9 m (most of the buildings have one or two storeys), no arcs in the building, no arc lintels, and low compressible or noncompressible ground (geology shows that rigid bedrock may exist at a depth less than a few meters).

5.2. *Development of vulnerability curves and functions*

According to Fig. 3, the next step in determining vulnerability and fragility curves is damage assessment for all theoretical buildings for different values of the horizontal ground strain between 0 and 10 mm/m. All of the empirical methods were then implemented in Mathematica software [22], and the damage level of each building was calculated for the different values of horizontal ground strain. The probability of damage in each damage class "$P(D_i)$" was then calculated with Eq. 3.

$$P(D_i) = \frac{N(D_i)}{n} \qquad (3)$$

$N(D_i)$ is the number of buildings in the damage class "D_i" and "n" is the total number of buildings (1000 in this example).

The vulnerability curve for the CF1 building type is the relationship between the mean damage and the horizontal ground strain and is calculated with Eq. 4.

$$\mu_D(\varepsilon) = \sum_{i=1}^{4} P(D_i) \cdot D_i \qquad (4)$$

$\mu_D(\varepsilon)$ is the mean of damages for the value "ε" of horizontal ground strain and $P(D_i)$ is the probability of damage in the class "D_i", as calculated with Eq. 3.

For example, we choose to present the results obtained with the mean damage method MD(ε) (Eq. 2) in Table 7 for the "CF1" building type. Results consist of the probability of damage in each damage class and the final row of this table is the mean damage for each value of horizontal ground strain.

Table 7. Results for the mean method and the "CF1" building type showing probability of damage in each damage class (D1 to D4) and mean damage in relation to horizontal ground strain (ε in mm/m).

D \ ε	0	0.5	1	1.5	2	2.5	3	3.5	4	4.5	5	5.5	6	6.5	7	7.5	8	8.5	>9
D_1	100	100	100	91	48	1	0	0	0	0	0	0	0	0	0	0	0	0	0
D_2	0	0	0	9	52	99	82	48	40	6	1	1	0	0	0	0	0	0	0
D_3	0	0	0	0	0	0	18	52	60	69	73	73	8	2	2	2	2	2	0
D_4	0	0	0	0	0	0	0	0	0	25	26	26	92	98	98	98	98	98	100
Total cumulate	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100
Mean of damage	1	1	1	1.1	1.5	2	2.2	2.5	2.6	3.2	3.2	3.9	4	4	4	4	4	4	4

The plot of mean damages is given in Fig. 6 and shows a discontinuous curve that is the consequence of using threshold values in all of the empirical methods. This result is hardly compatible with reality since damage should continuously increase with increasing horizontal ground strain. This assumption is also corroborated by the shape of all vulnerability functions developed in other fields, where a tangent hyperbolic function is often used [8]. To determine a continuous building vulnerability curve in agreement with the discontinuous curve previously plotted in Fig. 6, we fit the data to a tangent hyperbolic function according to Eq. 5

$$\mu_D(\varepsilon) = a[b + Tanh(c \cdot \varepsilon + d)] \qquad (5)$$

where $\mu_D(\varepsilon)$ is the mean of damages for a value "ε" of the horizontal ground strain, and *a, b, c,* and *d* are four coefficients that must be determined for each building type.

These parameters are not independent; two relations exist between them. According to Table 3, for horizontal ground strain equal to zero, there is no damage to buildings, and for horizontal ground strain greater than 9 mm/m, the mean damage to buildings is maximum and equal to four (greatest level in the damage scale).

Therefore, this leads to two boundary conditions detailed in Eq. 6, and only two parameters must still be determined. We used a nonlinear regression method to find the best values of these two parameters. The final continuous vulnerability curve for the “CF1” building type is shown in Fig. 6.

$$\begin{cases} \mu_D(0) = 1 \\ \mu_D(9) = 4 \end{cases} \quad (6)$$

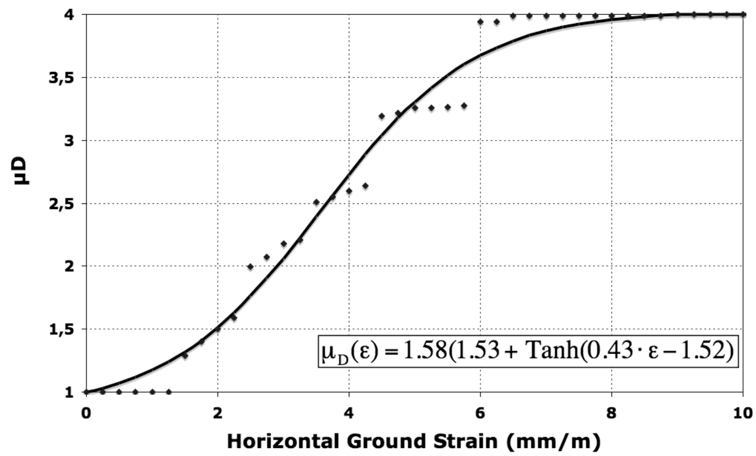

Fig. 6. Vulnerability function and curve for CF1 building type, built from Table 7.

The influence of the damage assessment method is investigated in Fig. 7. Results of the vulnerability curves for the MR2 building type (Table 5) obtained with each method show significant differences. In particular, the NCB [9] or Wagner and Schumann [10] method, NCBWD(ε), gives less damage than the other methods and this method is thus considered less conservative. The mean method MD(ε) logically gives a middle curve. Unless the user has scientific arguments for justifying one method by considering the special features of the studied case, it may be concluded that the mean method MD(ε) gives the most probable damage assessment.

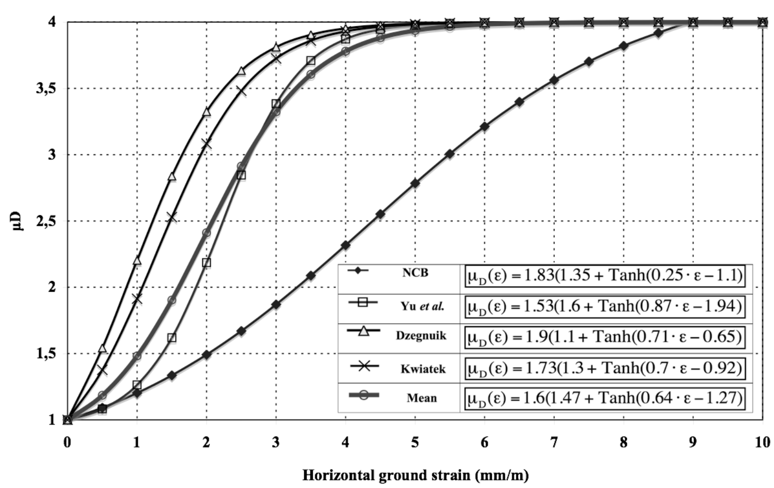

Fig. 7. Vulnerability curves for MR2 type by all methods.

5.3. *Development of fragility curves and functions*

Fragility curves are relations, for a given building type, between the probability of reaching or exceeding each damage level for different values of the horizontal ground strain "ε". These curves are directly obtained from the results in Table 7. The probability of reaching or exceeding one damage level is calculated from Eq. 7.

$$P(Damage \geq D_i) = 1 - P(Damage < D_i)$$
$$P(Damage < D_i) = \sum_{J=1}^{i-1} \frac{N(D_J)}{n} = \sum_{J=1}^{i-1} P(D_J) \quad (7)$$

$P(Damage \geq D_i)$ is the cumulative probability that the damage level exceeds the damage level "D_i", "$N(D_J)$" is the number of buildings in the damage class "J", and "n" is the total number of buildings (1000 in this example). $P(D_J)$ is the probability that the damage fall into damage class "D_J."

We attempted to find continuous, more realistic curves by using the lognormal distribution (Eq. 8) in agreement with mathematical functions regularly used in other fields (earthquake engineering [23],[24] and volcanic engineering [5]).

$$P[Damage \geq D_i \middle| \varepsilon] = \int_0^{\varepsilon} \frac{1}{\varepsilon\sigma\sqrt{2\pi}} \exp[\frac{-1}{2}(\frac{\ln(\varepsilon)-\mu}{\sigma})^2] d\varepsilon \quad (8)$$

In this equation, "$P(Damage \geq D_i/\varepsilon)$" is the probability of reaching or exceeding a damage level "D_i" for the value "ε" of the horizontal ground strain and "μ" and "σ" are the mean and standard deviation of ln(ε), respectively. The two parameters, "μ" and "σ", must then be determined for each building type and each empirical method used. We used a nonlinear regression method to fit the curve and find the best values for these parameters.

The resulting fragility curves and equations for the CF1 building type and for the mean damage method MD(ε) are shown in Fig. 8.

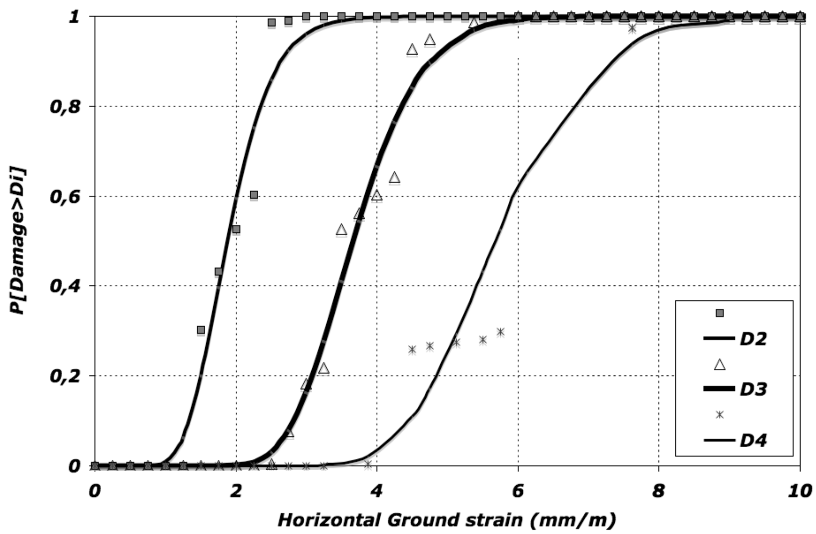

Fig. 8. Fragility data and curve for CF1 building type and the mean method.

6. Application and validation of the methodology for masonry buildings in Lorraine (France)

In France, the Lorraine region features large quantities of iron, salt, and coal deposits that were heavily extracted until the beginning of the 1990s (salt continues to be mined here). The presence of former iron mines raises many issues, including that of building vulnerability.

Between 1996 and 1999, five subsidence events occurred (two in the city of Auboué in 1996, two in Moutiers in 1997, and the last in Roncourt in 1999) which caused damage to more than 500 dwellings [1]. Many other cities and villages in this area may still be affected by this phenomenon.

Most of the buildings in Lorraine region can be classified into five types (Table 5). The methodology is applied to calculate the vulnerability curves with the mean method (Fig. 9a). Results show that the mean damage for a concrete building (CF1 type) is lower than that for a masonry building. Furthermore, comparison of vulnerability curves of the four types of masonry buildings show that the damages to unreinforced masonry (MR1, MR2, and MR3) are higher than those of confined and reinforced masonry buildings (MC1). In addition, comparison of the three types of unreinforced masonry buildings shows that the MR3 building type is more vulnerable than MR1 and MR2 because of higher values of building length. Finally, the very similar results for the two types MR1 and MR2 may be justified by the similarities of these two types, which differ only in their foundation parameter. Therefore, we can consider these two buildings types to be equivalent.

In order to validate the methodology and the final vulnerability curves, building damage observed in two cities (two subsidence events in Moutier and one subsidence event in Roncourt) are superimposed in Fig. 9b with the MR3 type (more vulnerable type) and the CF1 type (more resistant type). Results show significant similarities, and it may therefore be concluded that the vulnerability functions may be used to assess the building damage in other mining subsidence hazard areas.

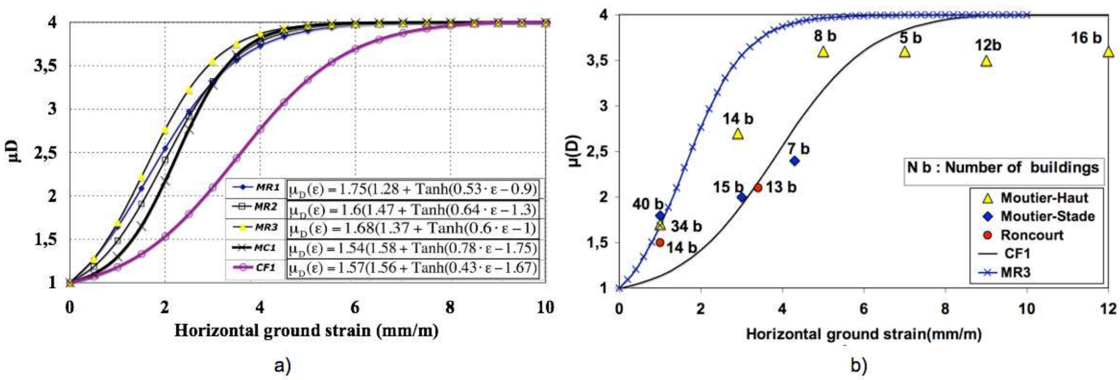

Fig. 9. a) Vulnerability curves for five types of buildings in Lorraine. b) Vulnerability curves and observed damage data in Lorraine.

7. Summary and Conclusion

This paper describes and applies a methodology for developing vulnerability and fragility curves using empirical methods already used for damage assessment. To do this, horizontal ground strain is chosen as the intensity criteria and a comprehensive typology is developed which is suitable for buildings in mining subsidence hazard areas. This typology is based on the most important building parameters in relation to the behavior of buildings affected by a subsidence event and is based on the criteria used in the empirical methods. Four parameters are defined: structural materials, building length, basements, and foundations, in addition to the building shape and symmetry. The typology is applied to buildings in the Lorraine region, where most buildings may be grouped into five classes.

The empirical methods used are the NCB [9] or Wagner and Schumann [10], Yu *et al.* [14], Dzegniuk *et al.* [15], and Kwiatek [16] methods. Because these methods are not easily comparable due to the use of different damage levels, different criteria, and different threshold values, the research presented in this paper required some modifications and a harmonization of these methods. Finally, all of the methods use four building damage levels, four building vulnerability classes, and 16 threshold values of the horizontal ground strain.

Vulnerability and fragility curves are based on the existence of variability concerning the behavior and the resistance of similar buildings. These curves are first obtained by a direct calculation of the damage level of

1000 buildings of the same type, and then are modeled by mathematical functions after a nonlinear regression. Vulnerability curves are modeled with a tangent hyperbolic function, and fragility curves are modeled with a lognormal distribution function. For both of these methods, two parameters must be calculated.

Results of the vulnerability curves show that the NCB [9] or Wagner and Schumann [10] method is significantly more conservative than the others are. Consequently, the choice of empirical method used to assess building damage and to develop the vulnerability curves appears to be debatable. A mean method is developed that corresponds to the mean value of the damage calculated with each method. Unless the user has scientific arguments for justifying one method considering the special features of the studied case, it may be concluded that the mean method gives the most probable damage assessment.

Application of the methodology to the five building types in Lorraine shows significant differences between masonry buildings and concrete buildings. Comparison of these results with observed damage induced by mining subsidence show significant similarities that validate the methodology.

In addition, fragility and vulnerability curves are a better compromise between accuracy and ease than existing empirical methods. Moreover, the mean damage method allows all of the empirical methods to be taken into account together and makes it possible to carry out necessary studies of the vulnerability of undermined cities. For completeness, vulnerability and fragility curves must still be compared with equivalent curves obtained by analytical methods that take into account the mechanical properties of the structure and the soil-structure interaction phenomena that occur during a mining subsidence.

8. Appendix.

8.1. Rating methods for the evaluation of building damages in a subsidence region

Table 8. Yu *et al.* [14] method (the number of points for each parameter is marked by (→); N = No damage, Ar = Architectural damage, F = Functional damage, St = Structural damage).

Foundations: Isolated footing (→ 1) ; Continuous footing (→ 4) ; Raft foundation (→ 8) ; Buoyancy foundation (→ 16)
Superstructure Material: Brick, Stone and Concrete (→ 4); Reinforced concrete (→ 4) ; Timber (→ 6) ; Steel (→ 8)
L/R ratio (L: length of the building and R: Radius of the subsidence): <0.1 (→ 8) ; 0.1-0.25 (→ 6) ; 0.26-0.5 (→ 4) ; >0.5 (→ 2)
H/L ratio (H: height of the building and L: length of the building): <1 (→ 8); 1-2.5 (→ 6); 2.6-5 (→ 4) ; >5 (→ 2)

Building classification																
Total Rating	7-10				11-20				20-30				30-40			
Vulnerability class	C_1				C_2				C_3				C_4			
Damage category	N	Ar	F	S	N	Ar	F	S	N	Ar	F	S	N	Ar	F	S
Thresholds value horizontal strain [mm/m]	<0.5	0.5-1.5	1.5-3	>3	<2	?	?	?	<3	?	?	?	<4	?	?	?

Table 9. The “Bhattacharya and Singh” method.

Building classification												
Brick and masonry structures/brick bearing walls /low rise structures												C_1
Steel and reinforced-concrete frame structures												C_2
Timber frame structures												C_3
Massive structures of considerable rigidity/central core design												C_4
Threshold values of horizontal ground strain for damage evaluation												
Damage category	Architectural				Functional				Structural			
Building class	C_1	C_2	C_3	C_4	C_1	C_2	C_3	C_4	C_1	C_2	C_3	C_4
Threshold values of horizontal strain	0.5	?	1	?	1.5-2	?	?	?	3	?	?	?

Table 10. Dzegniuk *et al.* [15] method (the number of points for each parameter is marked by (→)).

Building Length (m) : ≤10m (→ 4) ; 11-15 (→ 7), 16-20 (→ 11) ; 21-25 (→ 16) ; 26-30 (→ 22) ; 31-35 (→ 29) ; 36-40 (→ 37) ; >40m (→ 42)
Building solid shape: Regular, compact (→ 0) ; Little dismembered (→ 3) ; Well dismembered (→ 6) ; Regular, vast (→ 6) ; Dismembered, vast (→ 8)
Building foundation: On flat level, buildings with or without basement (→ 0) ; On uneven elevation, surface (→ 3) ; On uneven elevation, surface with partial basement (→ 6) ;As above but with a passage gate (→ 8)

Building ground foundation: Compressible (→ 0) ; Low-Compressible (→ 4) ; Uncompressible (→ 12)					
Building Structure: Rigid (→ 0) ; Low-Rigid (→ 4) ; Non-Rigid (→ 8)					
Existing protection for mining operation effects: Bolting (→ 0) ; Fractural bolting (→ 4) ; None (→ 6)					
Technical condition of the building: Good (→ 0) ; Average (→ 4) ; Bad (→ 12)					
Building classification					
Total Score	≥48	37-47	28-36	21-27	≤20
Vulnerability class	C_1	C_2	C_3	C_4	C_5
Horizontal ground strain	1.5 mm/m	1.5-3 mm/m	3-6 mm/m	6-9 mm/m	>9 mm/m
Damage category	Very slight or negligible	Slight damage	Moderate	Severe damage	Very severe damage

Table 11. The Kwiatek [16] method (the number of points for each parameter is marked by (→));

Building Length (m) : ≤10m (→ 2) ; 11-15 (→ 4), 16-20 (→ 7) ; 21-30 (→ 15) ; 31-40 (→ 20) ; 41-50 (→ 25) ; 51-60 (→ 30) ; 61-70 (→ 35) ; 71-80 (→ 40) ; 81-90 (→ 45) ; >91m (→ 50)
Building solid shape: Regular, compact block (→ 0) ; Regular, lying block (→ 2) ; Little dismembered, compact block (→ 4) ; Well dismembered, lying block (→ 6) ; Well dismembered, compact block (→ 8) ; Well dismembered, lying block (→ 10)
Building foundation: On flat level, buildings (→ 0) ; On uneven elevation, surface (→ 5) ; Foundation with carriage entrance, without cellar (→ 8)
Building ground foundation: Non rocky soils, except stones and rocks (→ 0) ; Backfilled ground (→ 4) ; Foundation on a layer of amortisement (→ 6) ; Stones and rocky soils, Except rock solid or slightly cracked (→ 10)
Building Structure: **A** - Foundation materials: Reinforced concrete (→ 0), Concrete (→ 2) ; Masonry brick (→ 3) ; Stones (→ 4) **B** - Walls of cellars: Concrete (→ 0) ; Masonry brick, locks or hollow concrete blocks (→ 1) ; Masonry stone, blocks hollow of reinforced concrete (→ 3) **C** - Floor of the lowest storey: Reinforced Concrete, *Ackermann*, with crowns made of reinforced concrete (→ 0) ; Concrete or reinforced concrete plan on steel beam (→ 1) ; Flooring with segments on steel beams, l/L>1/10 (l: width of segment) (→ 2) ; Flooring with segments on steel beams, l/L<1/10 (→ 4) ; Wood beamed (→ 3) ; Vault

without tie-beam, f / L> 1 / 5 (→ 4) ; Vault without tie-beam, f / L< 1 / 5 (→ 8) **D** – Lintels: Reinforced Concrete (monolithic or prefabricated) or on steel beams (→ 0) ; Bricks, plan (→ 2) ; Lintel arc, f/L>1/5 (→ 3) ; Lintel arc, f/L< 1/5 (→ 5) **E** – Other elements of building: Arcs in bearing walls, L> 1.5 m (without tie beam) f/L>1.5 (→ 4) ; Arcs in bearing walls, L> 1.5 m (without tie beam) f/L>1.5 (→ 8) ; Height of building blocks are different (→ 2) ; Level of floors are different (→ 3)					
Existing protection for mining operation effects: Building protected at all foundations and floors (→ 0) ; Building protected at the level of some foundations and floors (→ 2) ; Building protected at every floor (→ 8) ; Building protected in some floors (→ 10) ; Protection fragmented (→ 12) ; Without protection (→ 15)					
Technical condition of the building: Building State from naturally wear: Good (→ 0) ; Satisfactory (→ 1) ; Medium (→ 2) ; Bad (→ 3) ; Very Bad (→ 5) Pre damage of building: No degradation in the construction (→ 0); Cracks <1 mm (→ 2); 1<Cracks <5 mm (→ 5); 5<Cracks <15 mm or gap of out off plumb <25 mm (→ 8) ; 15<Cracks <30 mm or displacement or gap of out off plumb >25 mm (→ 12)					
Others: buildings that are not intended for permanent residence without heating (for example, box room, cowshed, barn) (→ -12) ; buildings for the temporary stay of people (workshops, garages) (→ -6) ; public buildings for the permanent or temporary residence of large groups of children, people, handicapped (→ 12) ; buildings with finishing equipment or sensitive to the influence of the exploitation (→ 6)					
Building classification					
Total Score	≥60	47-59	34-46	21-33	≤20
Vulnerability class	C_0	C_1	C_2	C_3	C_4
Horizontal ground strain	≤0.3 mm	0.5-1.5	2-3	4-6 mm	6-9 mm
Damage category	No damage scale is given in the method.				

8.2. *Comparison and synthesis of existing methods*

A comparison of the existing methods shows that their use and results raise some difficulties. The most important points can be summarized as follows:

- All of these methods use the horizontal ground strain parameter as an intensity criterion of the subsidence. This criterion is maintained for the development of the fragility and vulnerability curves.
- No building typology is clearly defined, and the methods use different criteria to assess the building resistance. Nevertheless, the length of the building always appears to be the most important

parameter. Other parameters concern building materials, existing reinforcements, shape of the building, building stiffness, and type of building foundations. Any building typology will have to be consistent with those criteria.

The damage scales used in the methods are different and may be compared with other scales used in mining subsidence (Pellisier *et al.* [17], Bruhn *et al.* [18] and Ji-Xian [19] (Table 12).

- The number of levels varies from three to six. We have chosen to use a four-level scale to develop the fragility and vulnerability curves: "D_1" for no damage or very slight damage, "D_2" for slight damage (D_1 and D_2 are considered to be architectural damage according to Bhattacharya and Singh [13]), "D_3" for appreciable or moderate damage (i.e., functional damage according to Bhattacharya and Singh [13]), and "D_4" for severe and very severe damage (i.e., structural damage according to Bhattacharya and Singh [13]).

- NCB [9] and the Wagner and Schumann [10] methods are very similar to each other. The evaluation of building damages in Joeuf city (a city in the Lorraine region) with these two methods shows that they give similar results, and thus can be considered as a single one [2].

- The abacus methods use a small number of criteria and allow damage to be assessed, whereas rating methods use a greater number of criteria to assess building resistance and give some threshold values of the horizontal ground strain. Moreover, the Yu *et al.* [14] method does not give the sixteen necessary threshold values in order to be truly operational (Table 8). Thus, the advantage that the rating methods seem more accurate (due to the greater number of criteria) is balanced by the lack of threshold values that are necessary to assess the damage level.

- The rating methods mainly define the vulnerability classes of buildings (four classes for the Yu *et al.* [14] and Bhattacharya and Singh methods and five classes for the Dzegniuk *et al*. [15] and Kwiatek [16] methods).

8.3. *Harmonization and modification of existing methods*

To develop fragility and vulnerability curves, the different empirical methods must be adapted to be both efficient and comparable. In particular, they must use the same number of damage levels, and the missing threshold values of the rating methods must be completed. For this purpose, we chose to develop unified vulnerability classes and common threshold values for the horizontal ground strain.

- *Harmonization of damage scales*

We selected a four-level damage scale ranging from D_1 (non damage or very slight damage) to D_4 (severe or very severe damage). First, this is comparable to the common scales used in the field of building vulnerability (D1 to D4 in the HAZUS method [4]). Second, it allows damages "D_1" that may be due to other causes than settlement (natural aging, in particular) to be distinguished from greater damage due to a subsidence. The levels of the selected damage scale are shown on Table 2.

- *Harmonization of vulnerability classes and threshold values of the horizontal ground strain for rating methods*

Regarding the number of vulnerability classes defined in the three rating methods (between 4 and 5), we suggest that rating methods be modified so that buildings may be classified into four vulnerability classes. This leads to groupings of two classes into a single one for two of the methods, and also makes the resulting vulnerability classes defined by the three methods comparable.

Indeed, a detailed analysis shows that the Kwiatek [16] method is very similar to the Dzegniuk *et al.* [15] method. Classes "C_0" and "C_1" defined in the Kwiatek [16] method may be considered equivalent to the class "C_1" of the Dzegniuk *et al.* [15] method. Classes "C_4" and "C_5" defined in the Dzegniuk *et al.* [15] method may also be considered equivalent to the class "C_5" of the Kwiatek [16] method. The final four classes obtained from these two methods are then compared with the four classes of the Bhattacharya and Singh method. A comparison of the threshold values of the horizontal ground strain (Table 8, Table 9, Table 10, and Table 11) shows that it is reasonable to assume that the four classes are comparable.

Determination of the damage level then strictly depends on the building vulnerability class and the intensity of the horizontal ground strain. Finally, their use requires 16 threshold values (4 damage levels x 4 vulnerability classes) to be defined. Unfortunately, half of these values are missing in the most complete method of Yu *et al.* [14] (Table 8). The missing values were chosen in agreement with the threshold values given by the other methods (Table 3). In particular, the threshold values of the Kwiatek [16] method, those that correspond to the maximum acceptable movement before significant damage, were used to define the values corresponding to the damage class "D_3". The original threshold values of the Dzegniuk *et al.* [15] method (Table 10) were used to define the threshold value of the damage class D_2 for the vulnerability class "C_2", the damage class D_3 for the vulnerability class "C_3," and the damage class D_4 for the vulnerability class "C_4".

Final threshold values were also adapted slightly to create a regular and logical increase in the values with increasing damage level or vulnerability class number.

- *Validation*

Table 3 is then compared with other methods or threshold values used by Ji-Xian [19], Boscardin and Cording [11], and Burland [12]. The Ji-Xian [19] method defines threshold values (Table 12) for buildings that may be classified into the third vulnerability class "C3," and the values are very close to those given in Table 3. Boscardin and Cording [11], and also Burland [12], developed abacus methods for unreinforced masonry buildings that may be classified into the first vulnerability class "C_1". In the case of mining subsidence, the value of the angle of distortion is mainly less than $2x10^{-3}$, and the horizontal ground strain appears to be the most important parameter. Threshold values of the horizontal ground strain are very similar to those given in the first column of Table 3.

Table 12. Building damage and threshold values of horizontal ground strain defined in the JI-Xian method ([19] and Table 3) for buildings equivalent to the vulnerability class C_3 of Table 3.

Threshold values of the horizontal ground strain	<2 mm/m	2-4 mm/m	4-6 mm/m	>6 mm/m
Damage category	D_1	D_2	D_3	D_4

Table 13. Building damage and threshold values of tensile building strain (assumed equal to the horizontal ground strain) as defined by Burland [12] and Boscardin and Cording [11] for buildings equivalent to the vulnerability class C_1 of Table 3.

Threshold values of the tensile strain	0-0.5 mm/m	0.5-0.75	0.75 -1.5	1.5- 3	>3
Damage category	Negligible	Very Slight	Slight	Moderate to severe	Very severe

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