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G-2015-139

December 2015

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La publication de ces rapports de recherche est rendue possible grâce au soutien de HEC Montréal, Polytechnique Montréal, Université McGill, Université du Québec à Montréal, ainsi que du Fonds de recherche du Québec – Nature et technologies.

Dépôt légal – Bibliothèque et Archives nationales du Québec, 2015.

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The publication of these research reports is made possible thanks to the support of HEC Montréal, Polytechnique Montréal, McGill University, Université du Québec à Montréal, as well as the Fonds de recherche du Québec – Nature et technologies.

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Metaheuristics as a solving approach for the infrared heating in the thermoforming process

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December 2015

Les Cahiers du GERAD

G-2015-139

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Abstract: Thermoforming process is a technique widely used in the plastic industry. This process involves three stages: i) sheet heating, ii) forming, and iii) cooling and solidification. A crucial problem occurring during this process is the non-uniform distribution of the energy flux intercepted by the plastic sheet during infrared heating. To circumvent the occurring of this phenomenon, the approach that we followed is first model the heat process as an optimization problem. Then, two meta-heuristic algorithms, simulated annealing and migrating bird optimization algorithms, are applied and compared so as to meet as much as possible the corresponding objective function. An experimental study is then conducted to evaluate the quality of the solutions produced by both algorithms. Results are presented and analysed.

Key Words: Infrared heating, migrating bird optimization, simulated annealing, thermoforming process.

Acknowledgments: This research was partially financed for Djamal Rebaine, Fouad Erchiqui and Issouf Fofana by the Natural Sciences and Engineering Research Council of Canada (NSERC), and, for Kahina Bachir Cherif, by the Group for research in decision analysis (GERAD) and NSERC.

1 Introduction

Sheet thermoforming is widely used in the plastic industry. This process involves three stages: i) reheat: the initial polymeric sheet is oven-heated to a softened state using radiative heat transfer, ii) forming: the heated sheet is deformed into the mold under the action of air flow, and iii) cooling and solidification: the polymeric sheet cools in the mold. Temperature and deformation distribution in the final container can be predicted using a robust modelling of the energy and momentum equations (see e.g. Wiesche et al. [8]).

When heated by infrared radiation, the plastic sheet is transformed from glassy into a rubbery state. This hot state combined with the gravity creates a non-uniform thickness distribution in the plastic sheet. Adequate optimization of the heating stage can improve significantly the mass distribution in the finished part. One effective way to achieve better uniform thickness distribution is to reduce the differences of energy intercepted and absorbed by the different areas of the thermoplastic sheet. An illustration is pictured by Figure 1, in which the colours represent the intensity of energy received by the thermoforming sheet:

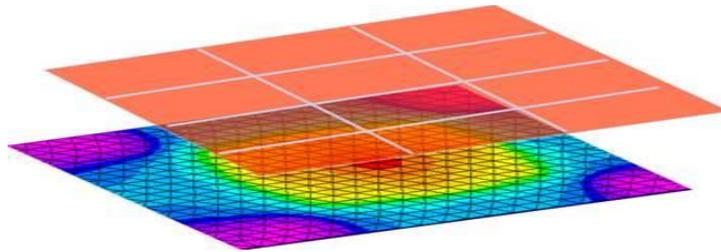


Figure 1: Distribution of energy received by a thermoplastic sheet from an infrared source

The classical methods of optimization of thermoplastics radiation have been the subject of many studies in the literature in finding the optimum solution of continuous and differentiable functions (see e.g. Boutaous et al. [1]). These methods are analytical in the sense that they make use of the techniques of differential calculus to locate optimum points. However, practical problems usually involve objective functions that are not continuous, nor differentiable. Therefore, the classical optimization techniques have a limited scope for those applications. One way to circumvent this difficulty is to discretize the problem so one can use iterative processes to rapidly produce reasonable solutions, but without the optimality guaranty.

The present paper is organized as follows. Section 2 introduces the problem we are considering. In Section 3, we present in an informal way the corresponding optimization formulation that we derived for the problem we are considering. In Section 4, we highlight two meta-heuristic algorithms *viz.* the simulated annealing and the migrating bird optimization algorithms. Section 5 first presents details on the implementation issues for both algorithms, and then results of the comparison study we conducted on the quality of the solutions produced by both algorithms.

2 Problem description

In the thermoforming process, the thermoplastic sheet is heated and formed in a mold through application of heat and pressure. The pressure is applied by introducing a vacuum, compressed air, or matched molds. Ceramic and quartz radiators are the main radiation sources used in an industrial scale thermoforming machines to heat plastic sheets. In the case of ceramic radiators, the entire mass of the system is maintained at a temperature of 570K–950K. On the other hand, quartz radiator systems use incandescent coils, including fused silica radiators with a coil temperature of 1200K, quartz glass radiators (1200K), and halogen radiators (2500K). Generally, the oven has a heater bank of n ceramic elements. The illustration of this process is pictured by Figure 2.

The incident radiation Q_i^j on cell i of the thermoplastic sheet surface S_i is calculated according to the geometric orientation of surface S_j and temperature t of oven heater cell j . View factors are required to obtain the energy for each area of the thermoplastic surface exposed to the infrared radiation. Assuming

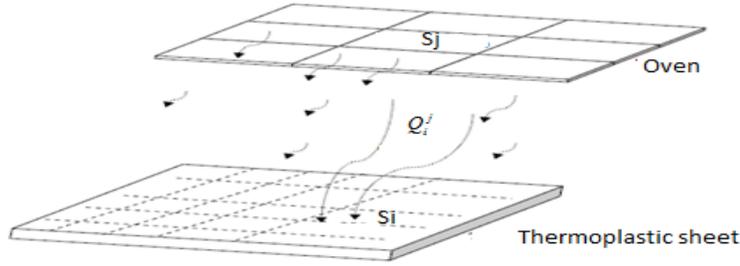


Figure 2: The thermoforming (heating) process

that surface S_j of the heater is separated by a transparent medium from surface S_i of the thermoplastic sheet, the amount of radiation Q_i^j leaving area S_j and reaching area of S_i is given by the following equation (see e.g. Erchiqui et al. [5]):

$$Q_i^j = F_{ij} \frac{S_j}{S_i} \sigma \bar{\epsilon} t^4. \quad (1)$$

Parameter F_{ij} denotes the view factor, $\bar{\epsilon}$ is the average source emissivity of the material. Parameter σ is the Stefan-Boltzman of value $5.67 \cdot 10^{-8} \text{ W/m}^2\text{K}$, and t represents the temperature from the temperature set τ assigned to cell j .

The view factors, given by the following

$$F_{ij} = \frac{1}{S_i} \int_{S_i} \int_{S_j} \frac{\cos \theta_i \cos \theta_j}{\pi \times r^2} dS_i dS_j, \quad (2)$$

are required to calculate the energy on the thermoplastic surface exposed to the infrared source. Note that, as illustrated by Figure 3, θ_i and θ_j are angles formed by the distance vector \vec{r} and the normal vector \vec{n}_1 (respectively \vec{n}_2). Parameter r is the distance separating surface dS_i and dS_j .

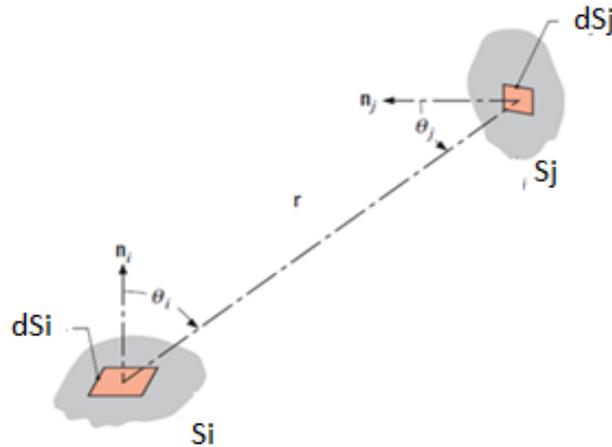


Figure 3: Geometric definition of the view factor

Note that the evaluation of the double integrals of (2) is estimated with the numerical Gaussian integration method (see e.g. Dhatt and Touzot [3]).

3 Statement of the optimization problem

Before proceeding into the modelling process, let us recall that the energy received by each cell of the thermoplastic sheet from a single heating cell is given by (1). The criterion that evaluates a uniform distribution

of the energy received by the thermoplastic sheet may be constructed as follows. The difference of energy received by two elements of the thermoplastic plastic sheet has an important influence on the quality of the final form of the product. Obviously, the smaller are the differences between these elements the better is the quality of the product. The goal is thus to make the elements of the thermoplastic sheet receive the same amount of heat from the heating elements. One way to achieve this goal is to minimize the standard deviation of the energy, compared to the mean, received by the cells of the thermoplastic sheet. To be more explicit, we proceed as follows. The expression of Q_i^j may be written, for a given temperature τ_k to choose from $\tau = (\tau_1, \dots, \tau_q)$, as

$$Q_i^j = F_{ij} \frac{S_j}{S_i} \bar{\varepsilon} \sigma \tau_k^4.$$

It follows that the total energy received by cell i of the thermoplastic sheet is

$$E_i = \sum_{j=1}^m Q_i^j.$$

If $\bar{x} = \frac{1}{n} \sum_{i=1}^n E_i$, then the goal is to minimize

$$\frac{1}{\bar{x}} \sqrt{\sum_{i=1}^n (E_i - \bar{x})^2}, \quad (3)$$

such that any cell of the heating oven receives exactly one temperature from set τ , and a temperature from τ may be used at most m times.

Let us first make the following point about the objective function. The above problem was the subject of the work presented in Erchiqui et al. [5], and Nahas et al. [6]. The objective function they studied is similar to the one presented in (3), but without the denominator \bar{x} of the fraction. The weakness of that function is that its minimization tends to favour the small values of the temperature set as they produce small values for that objective function. We recall that the goal is to find a small value of the standard deviation, but in comparison to its mean; the value of a standard deviation of data distributions is meaningful only when compared to its mean. In other words, what matters is the distribution of the values around their mean value, not the value of the standard variation on its own. This is the main reason the denominator appears in (3).

As stated, the above optimization problem is an extended version of the quadratic assignment problem. Indeed, the problem we are considering here may be viewed as seeking a mapping (and not a permutation as it is in the classical quadratic assignment problem) between the set of the temperatures and the set of the oven cells such that (3), a quadratic objective function, is minimized. It is known that the quadratic assignment problem is NP-hard (see e.g. Burkard et al. [2]). Therefore, the approximation approach is well justified as a solving method.

4 Metaheuristic approach

In this section, two meta-heuristic algorithms, simulated annealing and migrating bird optimization algorithms, are developed as a solving approach to distribute uniformly the energy intercepted by the material sheet.

4.1 The simulated annealing method

The simulated annealing (SA) approach is a technique that uses the analogy of a metal cooling until a minimum crystalline energy is reached. The goal is to bring a system from an arbitrary initial state to a state with a minimum possible energy.

In the implementation we designed we added two parameters. The first, the variable I , allows the exit from a given temperature if the solution is not improved after a certain number of successive iterations.

The second, parameter J , is another counter that permits the algorithm to stop if the current solution is not improved after a certain number of successive temperatures. The simulated annealing algorithm we implemented is resumed as below; for more details, see e.g. Pibouleau et al. [7].

1. Generate an initial solution S , and initialize T and $c(0 < c < 1)$.
2. Set the maximum number of iterations **iter-max**, the maximum number U of successive temperatures T the current solution is not improved, the successive number I of times the current solution is not improved for a given T .
 - a. Generate a new \acute{S} in the neighborhood of S and $\Delta f = f_S - f_{\acute{S}}$;
 - b. if $\Delta f > 0$ then $S \leftarrow \acute{S}$;
 else { - generate at random r in $[0,1]$;
 - if $r < \exp(-\Delta f/T)$
 $S \leftarrow \acute{S}$ };
 - c. Set $T = T * c$ if I is reached;
3. Repeat Step 3 until either **iter-max** or U is reached.

The neighbourhood utilized for the above version of SA is as follows:

Procedure Neighborhood {

1. Generate at random r in $[0,1]$;
 2. if ($r < 0.5$)

Select at random two temperature locations i and j from the actual solution, and then exchange the two corresponding temperatures;

else Select at random a temperature location i from current solution S , and choose at

random a temperature t from τ and assign it to cell i .
- } // end of the procedure

This technique of exploring the neighbourhood space allows a certain diversification. Indeed, if a solution cannot be improved with the permutation of the actual temperatures, then a new temperature from the temperature set is generated to replace a temperature of an arbitrary cell.

4.2 The migrating bird optimization method

The Migration Birds Optimization (MBO) was recently introduced by Ekram et al. [4]. Their method is inspired from the shape of the flight in ‘V’ of the migrating birds. The property of bird flights lies mainly in the energy conservation. Indeed, when a bird beats its wings, it generates a draft which will make the birds behind supplying fewer efforts to rise. The organization of the flight of birds is as follows: the bird at the head leads the group for a certain period, and spends more energy than the rest of the birds. When it gets tired, it moves to the back of the group, and one of the birds behind takes the lead. The idea behind the algorithm that one may derive works as follows. The leader is chosen according to the best performance. At first, we select the best solution among the set of the solutions generated previously as initial solutions. Then, two lists are created, a left and a right list, to form the V-shape of the flight as follows: the first solution is added to the left list, the second in the right list, and so on until all the solutions are placed. This way of placing the solutions is similar to that of birds. Indeed, the strongest bird is placed forward, whereas the least strong are placed behind, among others the old birds (see Figure 4 for an illustration). This way of arranging the solutions may lead in the search for the best solution to more promising regions and find the solution we are looking for in a reasonable laps of time.

As the method is quite recent, let us detail its description. For the sake of clarity, we first start mentioning analogies between MBO and natural bird migration. This makes it easy to justify the approach.



Figure 4: V-shape of bird flights

MBO	Flight of migrating birds
Number of solutions P	Nombre of birds
Number α of evaluated neighbors	Induced strength that is required
Number β of shared neighbors	Distance between birds
Number x of iterations	Flight time of the leader bird

The parameters defining the above algorithm are thus as follows:

- P = the number of initial solutions.
- α = the number of neighbor solutions to consider.
- β = the number of neighbors to share with the next solution.
- x = the number of iterations to perform before changing the leader solution, and
- K = the maximum number of iterations the algorithm executes.

The MBO algorithm mainly consists of four phases summarized as follows:

Step 1: This step initializes the parameters of the algorithm. First, it is recommended to choose P as an odd number in order to have the same number of solutions in the two lists in addition to the bird leading the flock. The number α of generated neighbors to evaluate may be interpreted as the necessary power of the flight. This power is inversely linked to the speed. With a bigger α , the birds flight at lower altitude, which helps to explore the neighborhood in details. By generating a large number of neighbors, we better explore the search space. Birds behind get less tired compared with those who are forwards, and thus save more energy. To respect this property in the MBO algorithm, we use the mechanism of services between the solutions. For the solutions at the back of the flock, through β the number of neighbors to share with the next solution, by generating fewer neighbors and by using the neighbors of the solutions in front, the algorithm can respect the property of energy conservation. Finally, the number x of iterations, before changing the solution linked to the leader at the head of the flock, represents the time spent by the leader of the group before its replacement.

Step 2: To improve the leading solution, α neighbour solutions are randomly generated. If the best neighbour solution is better than the leader, the latter is replaced by this solution, otherwise, it remains unchanged. The $(\alpha - 1)$ neighbour solutions not used by the leader solution are sorted in increasing order according to their objective values. Then, two groups, one is at the left and the other one is at the right, are formed as follows: the first and the second neighbour solution are added into the left and right lists, respectively, and so on. Let us note that each of these two sets is of cardinality β .

Step 3: The process of improvement progresses along the lists of the frock towards the tail. For a solution in the left list, we generate $(\alpha - \beta)$ randomly solutions in its neighbourhood. Then, these solutions are evaluated. The best solution is used to improve the current solution. The process is repeated until all the

solutions in the left list are evaluated and possibly improved. The same procedure is used for the solutions in the right list. This procedure is repeated x times without changing the leader.

Step 4: we proceed in this step to the change of the leader solution. The procedure is similar to what make the real birds: the bird spending most energy moves back behind for a rest, and another bird will take the lead. By analogy, the leading solution is alternately moved into the extremities of the left and the right line of both lists and the first solution of the list where the leader was placed is passed on as the new leader. With this mechanism, a solution which does not succeed in improving itself with its own neighbours is replaced by one of the neighbours of the previous solution, if the previous solution is naturally more promising. In this way, the region around the most promising solution will be explored more in details.

The steps followed an MBO algorithm may be then resumed as below:

Procedure MBO {

1. Fix P, α, β, x and K (the maximum number of iteration MBO executes).
2. Generate arbitrarily n solutions, and place them in a **V**-shape.
3. Improve the leader solution by generating and evaluating α of its neighbors.
4. Improve the solution in the V-shape by evaluating $(\alpha - \beta)$ neighbors with the β best solutions not used in the solution ahead.
5. Repeat x times Step 2 and 3.
6. Move the leader to the back of the group, and move one of the next solutions that are behind it to the leader position.
7. If K is not reached, repeat Step 2 until Step 6.

} // end of the procedure.

5 Experimental study

This section describes the experiments that we conducted to compare the two proposed algorithms (SA and MBO). We limited this comparison to the quality of the solutions they produced by setting the number of evaluated solutions to a maximum value of 10 000. The characteristics of the thermoforming device are as follows. The number of the oven heating cells is fixed to 36 (6×6) elements. The size of the heating cells is 1.2×1.8 mm. The size of the thermoplastic sheet is 1×1.6 mm, and the distance that separates the thermoplastic sheet and the heating oven is fixed to 1m. The average emissivity $\bar{\epsilon}$ of the thermoplastic sheet is set to 0.8.

The two algorithms are implemented in Matlab, version 7.12.0 (R2012a), and executed on an INTEL 2.0 GHz Dual Core processor with a RAM of 8192 MB capacity. Let us also mention that each execution of both algorithms is done on 10 instances generated arbitrarily. The values of the temperature set, we used in each simulation, are drawn from the interval [400, 900].

We first conducted experiments to tune the parameters of SA and MBO. Then, we compared the quality of both algorithms on several types of instances.

5.1 Parameters tuning

In this section, we present the study we conducted on tuning the different parameters related to the simulated annealing and the migrating bird optimization algorithms.

The parameters of the simulated annealing that we focused on are the initial solution, the initial temperature, the decrease of the temperature, and the number of iterations. Except the initial solution, our choice for the rest of the parameters is based on those utilized in Nahas et al. [5] as follows:

1. The initial temperature value T is set to 200.

2. The decrease of T obeys the following updating $T = T \times c$, where $c = 0.99$.
3. The number of successive iterations without an improvement on the objective for a given temperature is set to $I = 40$.
4. The number of successive temperatures without an improvement on the objective function is set to $U = 40$.

With regard to the initial solution, we tested the following rules:

- **Rule 1:** fix each cell to an arbitrary temperature from the temperature set.
- **Rule 2:** fix the cells to an arbitrary temperature from the temperature set.
- **Rule 3:** fix the cells to the minimum temperature from the temperature set.
- **Rule 4:** fix the cells to the maximum temperature from the temperature set.

The results we got are summarized by Table 1.

Table 1: Selecting an initial solution for the simulated annealing algorithm

Instances	Rule 1		Rule 2		Rule 3		Rule 4	
	Avg	Min	Avg	Min	Avg	Min	Avg	Min
1	0.0315	0.0301	0.0289	0.0265	0.0274	0.0265	0.0225	0.0215
2	0.0251	0.0240	0.0215	0.0210	0.0202	0.0198	0.0169	0.0155
3	0.0290	0.0271	0.0279	0.0279	0.0257	0.0240	0.0225	0.0215
4	0.0265	0.0232	0.0260	0.0252	0.0250	0.0221	0.0222	0.0201
5	0.0290	0.0265	0.0270	0.0270	0.0235	0.0230	0.0210	0.0210
6	0.0272	0.0241	0.0254	0.0254	0.0242	0.0232	0.0221	0.0215

We have generated six different instances to compare the performance of the above rules. The results we got are summarized in Table 1, and pictured in Figure 5. We observed that Rule 2 generates better results than the rest. Therefore, in the simulated annealing we implemented, the initial solution is generated by Rule 2.

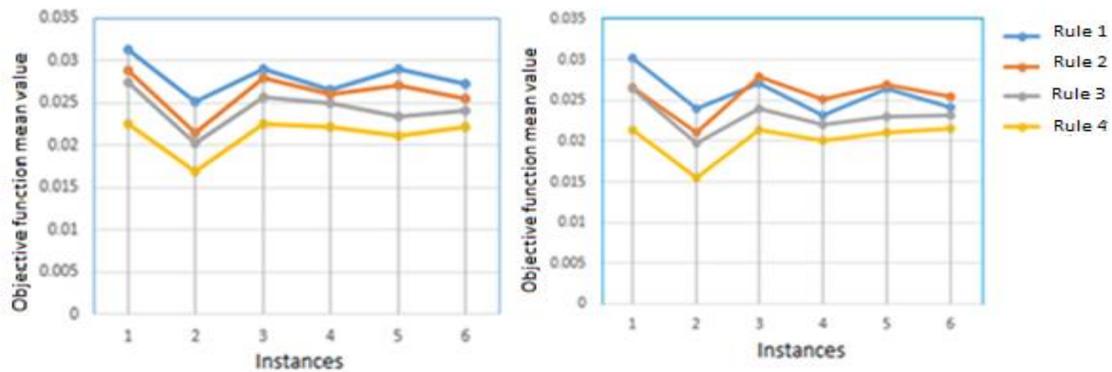


Figure 5: Minimal and average values of the objective function

With regards to MBO algorithm, we had to conduct a complete experimental study for the choice of the main parameters P, α, β, x and K . Let us first start with the initial solutions. Since MBO algorithm is a metaheuristic with a population, and in order to start the algorithm with a certain level of diversity, we generated initial solutions arbitrarily from a uniform distribution.

In Ekram et al. [4], it was recommended to set n to an odd number with a modest value. We tested our algorithm for n in the range 11 to 51. As shown by Figure 6a, we got better solution for $P = 51$. We tested α for modest values so as to explore a large number of solutions in the neighborhood of a solution. Our tests for $\alpha = 3, 5, 7, 11$, and 21, as resumed in Figure 6b, show that the results as much better for $\alpha = 7$.

Again, Ekram et al. [4] recommended choosing small values for β . The values we selected for our tests are $\beta = 1, 2, 3$, and 5 . The results we got, as resumed in Figure 6c, suggest the value $\beta = 3$. Again, it was recommended in Ekram et al. [4] to choose a modest value for x . We conducted a simulation for $x = 1, 2, 10$ and 20 . As resumed in Figure 6d, the results are better for $x = 10$. In the following, the abbreviations *avg* and *Std* mean average and standard deviation, respectively.

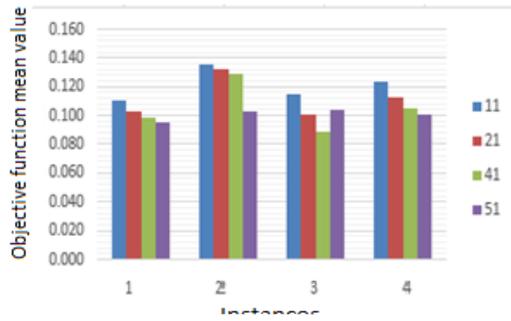
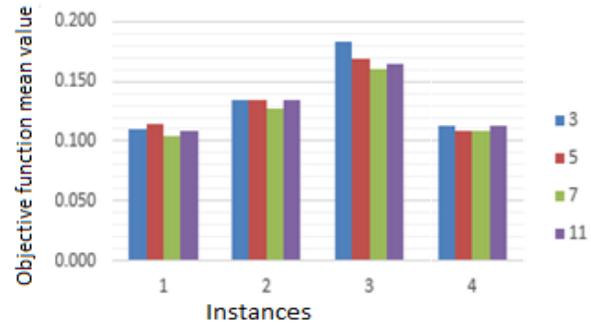
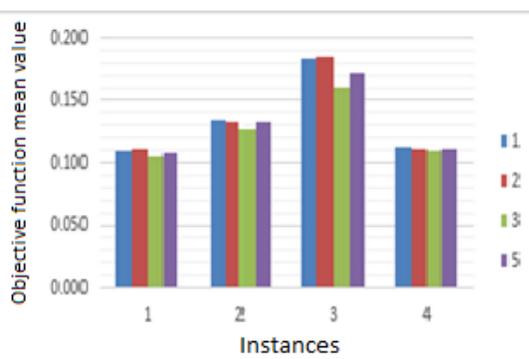
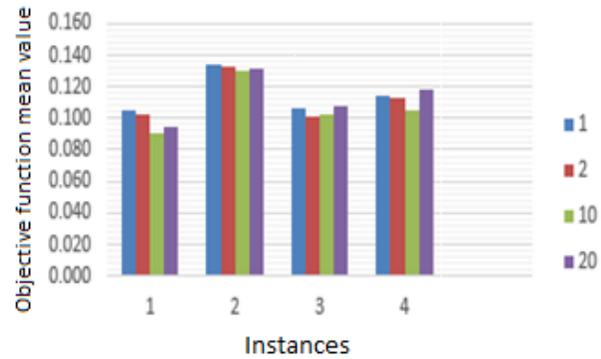
(a) Effects of variation of P (b) Effects of variation of α (c) Effects of variation of β (d) Effects of variation of x

Figure 6: Analysis to determine the value of the parameters MBO

5.2 Simulated annealing vs. migrating bird optimization

This section is devoted to the comparison of the two metaheuristics under study viz. simulated annealing and migrating bird optimization.

The different pairs (n, m) that we used, in the simulation we conducted to build the temperature set from which we select the temperatures to heat up the cell of oven, are follows:

Case 1. $n = m$: The number of temperatures generated is the same as the number of the cells in the infrared oven.

Case 2. $n > m$: The number of temperatures generated arbitrarily is less than the number of the cells of the infrared oven. We distinguished three subcases: $n = \frac{1}{4}m$, $n = \frac{1}{2}m$, $n = \frac{3}{4}m$.

Case 3. $n < m$: The number of temperatures generated arbitrarily is greater than the number of the cells of the infrared oven. We distinguished three subcases: $n = \frac{3}{2}m$, $n = 2m$, $n = \frac{5}{2}m$.

In the following, we present for the different cases, presented above, the results produced by SA and MBO and the best solution, in terms of selected temperatures assigned to the oven cells, generated by each approach, followed the corresponding energy flux received by the cells of the thermoforming sheet, respectively. We executed both algorithms 10 times for each set t of temperatures generated randomly.

Case 1. $n = m = 36$. The temperature set is as follow, and the results are as in Table 2:

$\tau = \{518, 589, 585, 619, 594, 489, 441, 826, 565, 732, 892, 506, 665, 443, 800, 851, 457, 842, 773, 537, 879, 834, 491, 847, 790, 463, 796, 691, 406, 460, 417, 760, 554, 403, 617, 605\}$

Table 2: Results produced by SA and MBO

	1	2	3	4	5	6	7	8	9	10	Avg	Std
MBO	0.059	0.059	0.058	0.057	0.057	0.053	0.055	0.058	0.057	0.053	0.057	0.002
SA	0.041	0.043	0.041	0.042	0.040	0.040	0.041	0.040	0.043	0.041	0.041	0.001

Table 3: Distribution of the temperatures produced by SA

892	790	403	403	732	892
665	403	403	403	403	403
403	403	403	403	403	403
403	403	403	403	403	403
851	403	403	403	403	796
892	406	403	403	441	892

Table 4: The corresponding distribution energy flux received by the thermoplastic sheet

968.9	1044.8	1073.2	1066.9	1037.7	985.1
1027.6	1107.1	1137.8	1130.9	1097.5	1038.2
1005.7	1081.4	1113.2	1108.9	1076.5	1017.4
996.2	1066.9	1096.4	1093.3	1065.3	1012.7
1019.0	1084.9	1108.0	1103.1	1080.7	1038.2
1001.6	1061.2	1078.0	1071.7	1055.4	1023.6

Table 5: Distribution of the temperatures produced by MBO

851	619	537	585	589	691
619	892	506	443	403	665
417	460	585	491	554	506
619	460	443	489	665	800
463	589	892	457	619	506
790	460	879	760	417	417

Table 6: The corresponding distribution energy flux received by the thermoplastic sheet

1165.8	1291.1	1362.9	1381.1	1349.2	1267.8
1312.7	1450.9	1523.9	1531.0	1478.8	1373.4
1352.9	1488.1	1554.8	1551.4	1485.3	1366.4
1348.6	1477.0	1537.8	1529.7	1459.1	1335.7
1325.7	1450.2	1508.1	1499.1	1429.0	1305.3
1250.2	1366.8	1419.5	1410.0	1343.9	1226.4

Case 2. $n < m$. We investigate here three subcases according to the value of n as a function of m .

Subcase 2.1. $n = \frac{1}{4}m = 9$. The temperature set is as follows, and the results are as in Table 7.

$\tau = \{459, 649, 880, 570, 693, 512, 776, 527, 653\}$

Table 7: Results produced by SA and MBO algorithms

	1	2	3	4	5	6	7	8	9	10	Avg	Std
MBO	0.060	0.058	0.058	0.058	0.054	0.055	0.056	0.054	0.058	0.055	0.057	0.001
SA	0.049	0.049	0.049	0.049	0.051	0.049	0.051	0.048	0.049	0.049	0.049	0.001

Table 8: Distribution of the temperatures produced by SA

880	880	459	459	880	880
776	459	459	459	459	459
459	459	459	459	459	459
459	459	459	459	459	512
880	459	459	459	459	776
880	776	459	527	693	880

Table 9: The corresponding distribution energy flux received by the thermoplastic sheet

1285,3	1402,2	1456,3	1457,7	1417,	1334,8
1385,7	1507,2	1561,7	1558,6	1509,3	1415,4
1372,4	1487,1	1539,0	1534,7	1483,2	1388,3
1358,7	1466,9	1514,8	1509,5	1460,0	1370,0
1380,2	1485,7	1527,5	1518,4	1470,9	1387,1
1352,7	1454,4	1491,0	1479,6	1435,8	159,89

Table 10: Distribution of the temperatures produced by MBO

857	808	716	808	808	539
808	463	463	857	879	857
673	673	808	448	879	463
808	673	857	853	857	857
539	808	463	716	808	463
539	673	448	673	673	673

Table 11: The corresponding distribution energy flux received by the thermoplastic sheet

1697.5	1897.3	2013.2	2031.9	952.93	1786.7
1881.0	2086.6	2196.1	2199.6	2101.7	1916.5
1906.9	2094.0	2182.9	2169.8	2064.0	1881.8
1869.1	2044.5	2124.2	2107.8	2006.8	1836.5
1810.5	1993.7	2085.4	2083.3	1996.8	1839.0
1676.7	1870.8	1983.3	2005.0	1939.8	1797.6

Subcase 2.2. $n = \frac{1}{2}m = 18$. The temperature set is as follows, and the results are as in Table 12.
 $\tau = \{796, 880, 728, 417, 825, 867, 740, 779, 772, 596, 728, 485, 753, 414, 538, 423, 448, 812\}$

Table 12: Results produced by SA and MBO algorithms

	1	2	3	4	5	6	7	8	9	10	Avg	Std
MBO	0.059	0.058	0.058	0.056	0.055	0.053	0.056	0.058	0.059	0.057	0.057	0.002
SA	0.045	0.044	0.044	0.044	0.045	0.043	0.044	0.044	0.044	0.045	0.044	0.001

Table 13: Distribution of the temperatures produced by SA

880	740	414	414	753	880
728	414	414	414	414	414
414	414	414	414	414	414
414	414	414	414	414	414
753	414	414	414	414	812
880	728	414	414	414	825

Table 14: The corresponding distribution energy flux received by the thermoplastic sheet

950.8	1025.6	1055.7	1053.5	1028.7	979.1
1019.1	1098.4	1130.4	1126.1	1095.2	1037.1
1000.0	1077.9	1112.1	1109.9	1078.5	1018.9
983.0	1058.8	1093.6	1093.6	1065.6	1010.5
1001.8	1075.9	1105.9	1102.6	1076.0	1026.0
997.1	1066.5	1087.8	1076.6	1047.3	1000.1

Table 15: Distribution of the temperatures produced by MBO

423	867	825	448	812	867
753	728	423	417	740	812
728	538	796	423	796	596
753	867	415	779	596	740
880	417	538	417	740	596
728	414	414	485	596	796

Table 16: The corresponding distribution energy flux received by the thermoplastic sheet

1509.2	1673.8	1764.1	1775.8	1714.0	1585.8
1679.6	1848.4	1930.8	1926.4	1845.1	1697.8
1719.0	1874.8	1941.3	1921.3	1828.3	1676.2
1714.6	1861.4	1919.2	1893.1	1797.8	1647.2
1694.8	1844.2	1905.7	1884.8	1794.9	1648.2
1600.6	1754.3	1825.9	1817.7	1740.2	1602.7

Subcase 2.3. $n = \frac{3}{4}m = 27$. The temperature set is as follows, and the results are as in Table 17:
 $\tau = \{596, 728, 485, 753, 415, 538, 423, 448, 812, 748, 558, 876, 417, 619, 591, 783, 798, 493, 645, 623, 723, 755, 778, 538, 740, 728, 481\}$

Table 17: Results produced by SA and MBO algorithms

	1	2	3	4	5	6	7	8	9	10	Avg	Std
MBO	0.059	0.060	0.059	0.061	0.056	0.057	0.056	0.060	0.061	0.059	0.059	0.002
SA	0.044	0.045	0.044	0.044	0.045	0.046	0.049	0.045	0.044	0.045	0.045	0.002

Table 18: Distribution of the temperatures produced by SA

876	778	415	415	798	876
755	415	415	415	415	423
415	415	415	415	415	415
417	415	415	415	415	415
783	415	415	415	415	798
876	723	415	415	723	798

Table 19: The corresponding distribution energy flux received by the thermoplastic sheet

1019.2	1101.8	1136.1	1133.9	1105.4	1048.6
1092.5	1178.9	1213.5	1207.8	1171.9	1105.7
1071.0	1154.1	1189.6	1184.9	1147.6	1079.7
1050.3	1131.1	1167.3	1165.1	1131.4	1067.6
1067.5	1148.3	1182.3	1179.7	1149.2	1090.6
1059.0	1137.8	1167.6	1162.3	1133.4	1079.0

Table 20: Distribution of the temperatures produced by MBO

755	448	728	755	645	623
740	591	619	728	723	596
753	728	755	493	723	448
740	481	753	596	558	728
493	415	876	778	493	485
485	481	740	728	415	778

Table 21: The corresponding distribution energy flux received by the thermoplastic sheet

1296.7	1448.7	1535.8	1548.8	1489.1	1364.9
1445.6	1605.4	1689.7	1692.5	1618.7	1478.8
1473.6	1623.8	1696.8	1689.2	1608.9	1468.2
1458.1	1601.8	1668.0	1655.0	1572.5	1434.1
1430.4	1575.7	1642.8	1629.9	1547.9	1411.0
1338.6	1482.2	1552.0	1544.8	1470.5	1342.3

Case 3. $n > m$. Again, we investigate three subcases according to the value of n as a function of m .

Subcase 3.1. $n = \frac{3}{2}m = 54$. The temperature set is as follows, and the results are as in Table 22:

$\tau = \{808, 853, 463, 857, 716, 448, 539, 673, 879, 883, 478, 886, 879, 643, 800, 471, 611, 858, 796, 880, 728, 417, 825, 867, 740, 779, 772, 596, 728, 485, 753, 415, 538, 423, 448, 812, 748, 558, 876, 417, 619, 591, 783, 798, 493, 645, 623, 723, 755, 778, 538, 740, 728, 481\}$

Table 22: Results produced by SA and MBO

	1	2	3	4	5	6	7	8	9	10	Avg	Std
MBO	0.063	0.059	0.059	0.059	0.057	0.057	0.057	0.064	0.059	0.062	0.059	0.003
SA	0.043	0.044	0.044	0.043	0.040	0.043	0.043	0.043	0.043	0.050	0.044	0.002

Table 23: Distribution of the temperatures produced by SA

886	853	415	415	800	886
716	415	415	415	415	418
415	415	415	415	415	415
415	415	415	415	415	417
883	415	415	415	415	783
886	558	415	415	716	883

Table 24: The corresponding distribution energy flux received by the thermoplastic sheet

1110,6	1203,4	1240,5	1234,2	1197,3	1130,4
1181,9	1277,9	1316,2	1308,3	1266,2	1191,8
1158,3	1248,2	1286,3	1280,3	1239,3	1165,9
1145,0	1229,1	1264,7	1260,4	1224,5	1158,1
1167,1	1248,1	1279,5	1275,6	1246,8	1190,6
1143,6	1221,1	1249,9	1248,9	1229,4	1185,1

Table 25: Distribution of the temperatures produced by MBO

645	858	800	755	481	858
558	539	623	886	728	418
539	493	463	723	853	728
876	858	591	867	596	880
716	478	783	886	728	596
673	591	417	812	596	853

Table 26: The corresponding distribution energy flux received by the thermoplastic sheet

1417.6	1588.8	1696.9	1732.7	1693.7	1581.9
1556.6	1746.7	1872.0	1922.6	1893.1	1781.4
1562.1	1744.4	1864.9	1914.9	1888.9	1783.6
1535.8	1704.4	1810.5	1847.8	1813.5	1707.2
1521.5	1687.1	1787.0	1815.4	1772.2	1659.8
1464.0	1631.0	1733.0	1762.7	1719.5	1607.1

Subcase 3.2. $n = 2m = 72$. The temperature set is as follows, and the results are as in Table 27:

$\tau = \{522\ 865\ 575\ 498\ 525\ 708\ 637\ 576\ 816\ 693\ 675\ 859\ 543\ 779\ 777\ 590\ 684\ 438\ 427\ 665\ 790\ 867\ 465\ 684\ 635\ 405\ 568\ 481\ 797\ 555\ 664\ 482\ 701\ 531\ 727\ 745\ 774\ 625\ 441\ 514\ 857\ 476\ 813\ 669\ 899\ 439\ 621\ 453\ 881\ 402\ 788\ 809\ 835\ 442\ 600\ 530\ 800\ 616\ 856\ 491\ 532\ 472\ 468\ 835\ 690\ 675\ 472\ 827\ 711\ 575\ 657\ 807\}$

Table 27: Results produced by SA and MBO

	1	2	3	4	5	6	7	8	9	10	Avg	Std
MBO	0.064	0.063	0.064	0.062	0.063	0.062	0.061	0.068	0.064	0.063	0.063	0.002
SA	0.041	0.042	0.043	0.043	0.041	0.041	0.041	0.043	0.044	0.041	0.042	0.001

Table 28: Distribution of the temperatures produced by SA

899	857	402	402	727	899
684	402	402	402	402	625
402	402	402	402	402	405
405	402	402	402	402	405
865	402	402	402	402	816
899	657	402	402	568	881

Table 29: The corresponding distribution energy flux received by the thermoplastic sheet

1422.2	1542.1	1590.3	1582.6	1535.5	1450.2
1523.6	1648.6	1699.1	1690.3	1638.0	1544.5
1494.5	1613.1	1664.3	1658.4	1607.3	1514.1
1469.9	1582.7	1632.0	1627.0	1578.5	1489.2
1497.4	1609.2	1653.2	1644.4	1596.8	1511.7
1483.6	1593.8	1633.0	1621.0	1575.9	1497.2

Table 30: Distribution of the temperatures produced by MBO

865	526	701	850	723	882
563	742	701	829	744	565
668	827	837	680	593	850
504	444	563	847	912	847
847	755	882	593	593	865
526	608	755	829	659	444

Table 31: The corresponding distribution energy flux received by the thermoplastic sheet

1924.0	2153.3	2282.2	2292.9	2187.9	1987.2
2141.6	2390.0	2530.1	2545.8	2439.3	2228.2
2200.5	2437.8	2568.8	2583.1	2484.5	2285.6
2195.8	2415.7	2531.0	2538.0	2443.0	2256.2
2153.0	2362.0	2467.4	2469.4	2375.9	2195.3
2018.2	2217.2	2318.5	2322.5	2235.2	2063.3

Subcase 3.3. $n = \frac{5}{2}m = 90$. The temperature set is as follows, and the results are as in Table 32:
 $\tau = \{853\ 463\ 857\ 716\ 448\ 539\ 673\ 879\ 883\ 478\ 886\ 879\ 643\ 800\ 471\ 611\ 858\ 796\ 880\ 728\ 417\ 825\ 867\ 740\ 779\ 772\ 596\ 728\ 485\ 753\ 415\ 538\ 423\ 448\ 812\ 748\ 558\ 876\ 417\ 619\ 591\ 783\ 798\ 493\ 645\ 623\ 723\ 755\ 778\ 538\ 740\ 728\ 481\ 459\ 649\ 880\ 570\ 693\ 512\ 776\ 527\ 653\ 750\ 846\ 880\ 674\ 469\ 474\ 529\ 821\ 527\ 807\ 522\ 865\ 575\ 498\ 525\ 708\ 637\ 576\ 816\ 693\ 675\ 859\ 543\ 779\ 777\ 590\ 684\ 808\}$

Table 32: Results produced by SA and MBO algorithms

	1	2	3	4	5	6	7	8	9	10	Avg	Std
MBO	0.061	0.061	0.063	0.060	0.057	0.059	0.058	0.060	0.060	0.061	0.060	0.002
SA	0.042	0.043	0.043	0.042	0.042	0.043	0.042	0.043	0.043	0.044	0.043	0.001

Table 33: Distribution of the temperatures produced by SA

886	415	415	415	796	886
693	415	415	415	415	527
415	415	415	415	415	415
415	415	415	415	415	415
880	415	415	415	415	858
886	693	415	415	448	886

Table 34: The corresponding distribution energy flux received by the thermoplastic sheet

1135,7	1233,0	1272,4	1265,9	1227,2	1157,9
1204,8	1305,5	1347,4	1341,3	1299,7	1224,9
1179,5	1273,8	1315,8	1313,2	1274,9	1203,3
1169,4	1257,0	1295,4	1293,7	1260,7	1197,4
1199,0	1281,8	1312,4	1306,5	1277,1	1222,9
1182,5	1259,5	1282,0	1271,2	1243,7	1197,2

Table 35: Distribution of the temperatures produced by MBO

776	859	463	645	807	469
716	821	853	728	858	708
753	596	723	783	527	645
883	867	716	880	779	649
880	538	674	653	527	543
693	418	740	417	576	825

Table 36: The corresponding distribution energy flux received by the thermoplastic sheet

1817.8	2021.3	2123.8	2118.3	2018.1	1844.7
2013.6	2235.8	2349.0	2347.9	2246.7	2064.9
2040.7	2261.8	2381.6	2394.8	2310.6	2141.6
2008.6	2232.9	2365.0	2395.1	2325.1	2162.6
1963.1	2196.7	2341.9	2383.0	2315.8	2146.7
1837.7	2064.6	2208.9	2252.6	2188.0	2019.8

Table 37 summarizes the results obtained from the execution of two algorithms through the instances we have generated. We note that, on the seven generated instances, the best results are produced by the simulated annealing algorithm. Nevertheless, some remarks connected to the average and standard deviations of the generated results are to be made:

1. For the seven instances we have studied, the method of SA generates by far better results than MBO in terms of average and standard deviation for the values generated for the objective function.
2. The discrepancy between the cells in terms of energy received by the thermoplastic sheet is significant for MBO than for SA.
3. Even though this was not our primary goal for the comparison we conducted, we noticed that the running time of SA to generate a solution is much better than that of MBO, and
4. The convergence of MBO towards the best solution is very slow compared with SA.

Table 37: Results produced by SA and MBO algorithms for all the cases

N	MBO			SA		
	Min	Avg	Std	Min	Avg	Std
9	0.054	0.057	0.001	0.048	0.049	0.001
18	0.053	0.057	0.002	0.044	0.044	0.001
36	0.053	0.057	0.002	0.041	0.041	0.001
54	0.057	0.059	0.003	0.043	0.044	0.002
72	0.061	0.063	0.002	0.041	0.02	0.001
90	0.058	0.060	0.002	0.042	0.043	0.001

We conclude not only from the above results related to the values of the objective function, but also by visualizing the distributions of the energy intercepted by the thermoplastic sheet generated by both algorithms, resumed by the above tableaux, that the simulated annealing algorithm clearly outperforms the migrating bird emigrating optimization method.

6 Conclusion

The study we conducted in this paper allowed us to adapt the MBO method for the resolution of a problem of thermoforming modelled in the form of a problem of combinatorial optimization, in this particular case, the problem of the quadratic assignment problem. Then, we conducted an extensive experimental study by first tuning the different parameters linked to SA and MBO, and by comparing the quality of the solution produced by these two algorithms.

This study reveals that the quality of the solutions generated by SA outperforms those of MBO in all the cases. Even from the running time point of view, this conclusion is still the same.

For further research, it would be interesting to design much more involved heuristics to generate an initial solutions and see to what extent this may influence not only the quality of the final solution, but also the convergence of the simulated annealing method. Different structures of neighborhood are also a good research avenue to improve the performance of SA.

On the other hand, we have suggested, through the expression of the objective function, another way of capturing the distribution of the energy flux received by the thermoforming sheet. Even though we improved the quality of this distribution, compared to the results generated by SA in Nehas et al. [6], we still observe significant discrepancies between cells as regards to the energy flux received by the thermoforming sheet. It would be of interest to improve the quality of this distribution by developing other objective functions. This avenue of research is in progress.

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