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Characterizing the Suspension Anisotropy of a Computerized Foucault Pendulum

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Abstract **-** All unsustained physical pendula undergo various types of damping processes which make them irreversible quasi-periodical devices. Their recent use as instruments for detecting cosmological micro-anomalies requires the determination of system anisotropy with the utmost precision, notwithstanding an inherent variance due to the strictly aperiodical behavior. An image processing algorithm has been developed for analyzing a camera monitored Foucault pendulum. This greatly improves precision in the determination of swinging azimuth, precession angle, swinging amplitude and swinging period. To the authors' knowledge, the very weak anisotropy of a long Foucault pendulum has been, for the first time, experimentally characterized in terms of the zero-amplitude swinging period plus a conservative wave in period and amplitude.

Keywords**:** Physical pendulum, Foucault pendulum, instrumentation, anisotropy, irreversibility, image processing

1. Introduction

The simple pendulum is idealized in practically all textbooks as a harmonic oscillator obeying conservative Hamilton's mechanics. However, the mere presence of some inevitable damping in a physical pendulum prevents the trajectory in phase space from being closed and periodical. To address the damping phenomenon with classical conservative perturbation methods, the phase space trajectory is made artificially closed by making the energy loss over one half-cycle stored in some imaginary potential reservoir, and then given back to the pendulum in a time reversal process for the next half-cycle, thus preserving the Hamiltonian description (Minorsky, 1962). The Foucault pendulum has been thoroughly investigated by Nobel Prize Kamerlingh Onnes (1879) in his dissertation, where it was idealized as a two-degrees-of-freedom harmonic oscillator. Kamerlingh Onnes was the first to apply the perturbation methods of celestial mechanics to the spherical pendulum. His main contribution was to show, theoretically and experimentally, that the natural tendency of the spherical pendulum to generate elliptical orbits after a few minutes of operation was largely due to suspension anisotropy. Some 200 years after Foucault, the French engineer and Nobel Prize Maurice Allais (1959, 1999 and Web-1) designed the so-called paraconical pendulum, with a rolling ball suspension. Allais characterized the amount and orientation of pendulum anisotropy by the initial rate of increase of the ellipse minor axis. Beside suspension anisotropy, he detected an unidentified, variable source of anisotropy which he attributed to space, since it was correlated with the motion of particular bodies in the solar system.

Much concern about the Foucault pendulum as a scientific instrument has been revived in the last 50 years. See Verreault (2013) for a short review. However, researchers are facing several drawbacks due to the irreversible, non-reproducible nature of the experiments. Namely, physical pendula show a non negligible 3rd degree of freedom (Verreault, 2011) in the form of a torsion about the suspension wire or rod axis (a sort of macroscopic spin degree of freedom), which takes part in the repartition of energy and momentum. Moreover, every physical pendulum with swinging amplitudes of a few centimetres undergoes prevalent aerodynamic damping with different power loss parameters for each degree of freedom. Finally, nonlinear coupling can also cause conservative energy transfer between the degrees of freedom.

In view of the recent interest in the spherical pendulum as a cosmological anisotropy measuring instrument, it is the purpose of this paper to show how, by using appropriate image processing algorithms, and irrespectively of the various dissipative processes, 1° a conservative energy transfer between the pendulum proper modes can be experimentally demonstrated; 2° the zero-amplitude swinging period can be precisely determined for each azimuth and used to characterize the pendulum anisotropy.

2. Experimental and Theoretical Background

The data used for this research originates from an 8 m Foucault pendulum set up in Gifu, Japan, for the solar eclipse of July 22, 2009 (Web-2). The experimental site consisted of white concrete bunker 15m x 13m x 8.5m on the University campus. The suspension rig was clamped to a 19mm U-bolt emerging from a ceiling concrete beam. A Watec 902-H2 video camera was horizontally fixed to the suspension rig so as to get, though a pair of mirrors at 135°, a parallax-free vertical view of the pendulum bob. The 18-hour continuous recording at 30 frames per second fills up a 1.5 terabyte HDD. The positions of the bob and of the reference alidade are made visible through a set of retro-reflecting markers which are illuminated from a small spotlight also near the suspension rig.

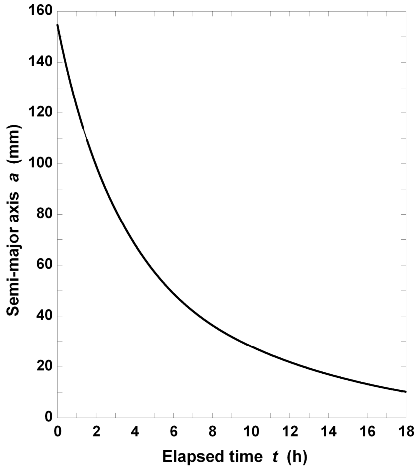
In order to achieve the maximum available precision from the data processing, proprietary software has been developed. In a pre-processing phase, a listing of the coordinates of the retro-reflecting markers is made. Then a quarter of ellipse is fitted to each quarter of a cycle, in order to determine the major and minor axes twice per cycle. Moreover, two period calculations per cycle are made by comparing the times of passage at the end of the major and minor axes between consecutive cycles. To make those period measurements comparable with those of the literature, which typically are averages over ~100 cycles, a running mean has been applied over 85 cycles troughout the data.

From the theory standpoint (Kamerlingh Onnes), the different longwise and crosswise elastic properties of the rig and beam assembly generate two distinct periods for swinging directions parallel and perpendicular to the beam, hence linear anisotropy which manifests itself as a tendency for the rectilinear oscillations to degenerate into elliptical orbits. A complete cycle of period values should thus be observed over a 180° azimuth span. Since that azimuth interval is normally scanned in half the Foucault period due to the Foucault precession, it is expected that the anisotropy features shall be expressed as a Fourier series with the half-Foucault period (20.68 h) for the fundamental.

Since all sorts of external perturbations, particularly during solar eclipses, also tend to induce elliptical orbits, it is important to precisely characterize the amount attributable to suspension anisotropy. For this work, only the evolutions of the swinging amplitude and the swinging period over the 18-hour duration of the experiment have been considered.

3. Results and discussion

Figure 1 shows the evolution of the amplitude. The actual graph consists of 19 803 experimental points which are determined with an average accuracy of 15 µm rms. This provides enough information to determine not only the viscous and aerodynamic damping parameters, but also a 3-harmonic Fourier series at the half-Foucault fundamental period because of the energy exchange between the major and minor axes when the latter grows up to suspension anisotropy. It can be verified that the maximum of the minor axis of Figure 2 coincides with the minimum of the fundamental wave of the major axis. Here, it is worth noting that the huge amount of redundant information present in video imagery allows unprecedented precision in the determination of the pendulum characteristics.



$$a = a_0/(\exp(\alpha t)+\beta t) + a_1\cos(\phi_1+2\pi t/20.68) + a_2\cos(\phi_2+2\pi t*2/20.68) + a_3\cos(\phi_3+2\pi t*3/20.68)$$

	Value	Error
a_0 (mm)	154.356	0.003
$1/\alpha$ (h)	6.6517	0.0006
$1/\beta$ (h)	10.065	0.004
a_1 (mm)	1.307	0.002
ϕ_1 (rad)	1.392	0.002
a_2 (mm)	0.3049	0.0006
ϕ_2 (rad)	6.011	0.002
a_3 (mm)	0.0386	0.0003
ϕ_3 (rad)	4.809	0.006
Residuals (mm)	0.015	

Fig. 1. Swinging amplitude as a function of time. The available information allows the significant determination, in addition to the viscous time constant α^{-1} and the aerodynamic "time constant" β^{-1}, of a 3-harmonics periodical part describing the energy exchange between the minor and major axes according to the anisotropy tendency to generate a minor axis wave at half the Foucault precession period (i.e. 20.68 h in Gifu).

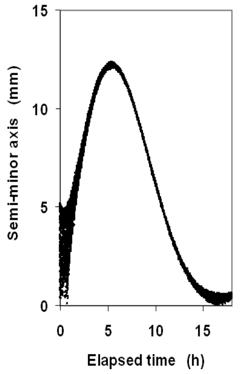
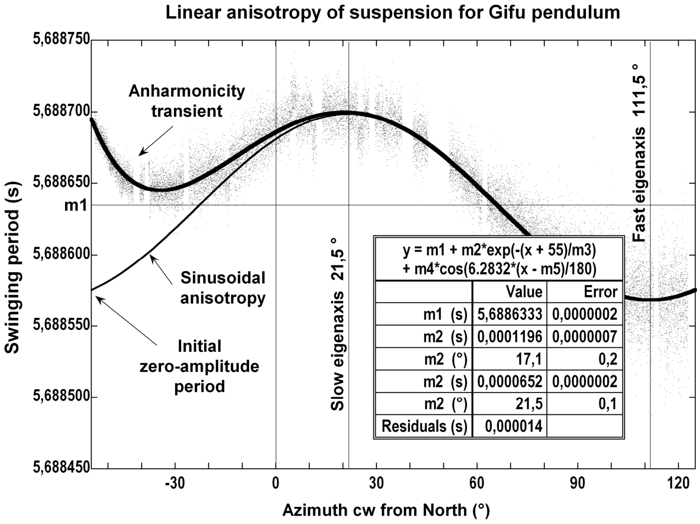


Fig. 2. Evolution of the minor axis during the 18-hour experiment in Gifu.

Figure 3 shows the experimental results for the evolution the swinging period as a function of swinging azimuth. Again, the 19 803 data points show an average residual of 14 µs, so the amount of available information clearly separate two contributions. A transient addition to the ideal zero-amplitude period is well identified, with a damping constant of 17.1°. The zero-amplitude period exactly fits a sine wave within the experimental errors, which verify the theory for linear anisotropy (a possible 24-hour temperature perturbation cycle did not materialize, due to overcast skies during the whole experiment).

4. Conclusion

The combined use of video imagery and proprietary software involving piecewise curve fitting over terabyte data stream has allowed unprecedented precision in the determination of the Foucault pendulum parameters. To the authors' knowledge, the weak suspension anisotropy of a long Foucault pendulum has been experimentally determined for the first time in terms of period measurements with sub-microsecond precision. One can then very exactly predict the amount of precession rate attributed to linear anisotropy of suspension, so that any amount there upon will have to be attributed to external perturbations.



y = m1 + m2*exp(-(x + 55)/m3)
+ m4*cos(6.2832*(x - m5)/180)

	Value	Error
m1 (s)	5,6886333	0,0000002
m2 (s)	0,0001196	0,0000007
m2 (°)	17,1	0,2
m2 (s)	0,0000652	0,0000002
m2 (°)	21,5	0,1
Residuals (s)	0,000014	

Fig. 3. Linear anisotropy of suspension for the Gifu pendulum, expressed as a pure sine wave for the swinging period in terms of the swinging azimuth. The swinging period includes a transient contribution due to anharmonicity and proportional to the square of the swinging amplitude.

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Web-2: http://www.youtube.com/watch?v=vsygRYfubVY/ consulted 11 April 2013.