An optimization model to maximize energy generation in short-term hydropower unit commitment using efficiency points

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Abstract

This paper presents a linear mixed-integer formulation to solve the short-term hydropower unit commitment problem. It uses the pair of maximum efficiency points of water discharge and the power produced at the maximum storage for all possible combinations of turbines. The goal is to maximize total energy production for all periods. The objective function is calculated using the correction between the power produced at the current volume and the maximum storage and penalizes unit start-ups. Constraints on the maximum number of turbine changes are imposed to find a viable solution. Computational results are reported for 65 instances with two powerhouses of five turbines each located in the Saguenay-Lac-St-Jean region of the province of Quebec in Canada.

Keywords: `Hydro unit commitment, short-term hydropower optimization, mixed integer linear programming`

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Nomenclature

The sets are:

$t \in \{1, 2, 3, .., T\}$: set of periods.

$c \in \{1, 2, 3, .., C\}$: set of powerhouses.

$l \in \{1, 2, 3, .., U^c\}$: et of powerhouses upstream of each powerhouse c.

$j \in \{1, 2, 3, .., J_t^c\}$: set of turbines associated to period t and powerhouse c.

$b \in \{1, 2, 3, .., B^c\}$: set of combinations of each powerhouse c.

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$k \in \{1, 2, 3, .., K^b c\}$: set of points (the maximum and the adjacent points) associated to powerhouse c and combination b.

The parameters are:

$P^c_{k,t}$: power output of powerhouse c at period t and point k (MW)

$q^c_{k,t}$: water discharge of powerhouse c at period t and point k (m^3/s).

δ^c_t: inflow of powerhouse c at period t (m^3/s).

β: conversion factor from (m^3/s) to (hm^3/h).

θ^c: estimated energy losses from maximum storage (MW) at powerhouse c.

ϵ^c: start-up penalty of turbine (MW) at powerhouse c.

N^c_{max}:maximum number of start-ups for powerhouse c.

V^c_{max}: maximum volume of reservoir c (hm^3).

v^c_{ini}: initial volume of reservoir c (hm^3).

v^c_{final}: final volume of reservoir c (hm^3).

Δt: thw duration of period t (h).

$$A^c_{t,k,j} = \begin{cases} 1 & \text{if the turbine } j \text{ of powerhouse } c \text{ at the point } k \text{ is activated at period } t \\ 0 & \text{otherwise} \end{cases}$$

The decision variables are :

$$y^c_{k,t} = \begin{cases} 1 & \text{if the point } k \text{ is chosen at period } t \text{ for powerhouse } c \\ 0 & \text{otherwise} \end{cases}$$

$$z^c_{j,t} = \begin{cases} 1 & \text{if the turbine } j \text{ of powerhouse } c \text{ is started at period } t \\ 0 & \text{otherwise} \end{cases}$$

v^c_t: volume of the reservoir of powerhouse c at period t (hm^3).

d^c_t: water spillage at powerhouse c and period t (m^3/s).

1. Introduction

1.1. Motivation

Hydropower is the third source of renewable energy in the world according to the International Energy Agency (IEA) in 2017 [1]. Electricity producers seek to manage their production either by maximizing the energy production or minimizing the operational cost. Managing hydroelectric systems is complex and requires different optimization processes. Long-term optimization models are used to determine the future production potential for a few years of the planning horizon and take into account the uncertainty of the inflows [2, 3]. Medium-term models are used to define the quantity of water available in the reservoir for hydropower production for a weekly scheduling horizon [4, 5]. Short-term models aim at defining the optimal strategy of the daily production by dispatching the quantity of water between the turbines in order to maximize the energy produced using the power production function or to minimize costs [6].

This paper focuses on the short-term model which determines for each powerhouse and for each time stage the optimal water discharge, the volume of the reservoirs, and the state of each unit (on or off) while respecting some constraints. For the short-term problem, these constraints usually involve water balance constraints, energy demand, water discharge constraints, reservoir limits and start-ups of the units [7].

1.2. Literature review

The study of the unit commitment problem has taken place during the past few decades. Several methods and algorithms can be used to solve this problem: dynamic programming [8, 6], Benders decomposition [9], heuristic methods like genetic algorithms and particle swarm optimization [10, 11] and sequential quadratic programming [12] among others. Many unit commitment problems are formulated by Mixed Integer Linear Programming (MILP). The advantage of MILP is that the discrete nature of the state of the units can be modelled using integer variables [13]. In addition, MILP can allows large size scheduling problems in energy power systems [14].

Because the hydroelectric production function are non-linear and non-convex, different techniques are used to handle this non-linearity. In [15] a MILP was proposed to solve the hydropower unit commitment problem in order to minimize the operational costs. In this model, the decision variables were the start-up and shutdown of the units and the water releases. The formulation of the problem considers the variation of the net head water and the non-linearity of the production function. This non-linearity was accounted for with a three-dimensional interpolation technique. The model was tested on a real case in China with one reservoir and 32 heterogeneous generating units. Another formulation in [16] uses MILP to maximize power efficiency. The problem was split into two phases. The first phase was a preprocessing based on unit commitment and a piecewise linear generation function to define the water discharge and the total powerhouse generation, considering the maximum discharge bound, the efficiency curves and restricted operating zones for each unit. In the second phase, a MILP formulation with fewer binary variables was built based on the preprocessing phase in order to maximize the final storage energy.

In [17] a unit commitment problem with head dependent reservoir was developed to find the optimal scheduling of a multiunit pump-storage hydropower system. A mixed integer linear model was formulated to solve the problem. The continuous variables are the water discharge, the volume of the reservoir, the produced power, the water spillage and the binary variables are the start-ups and shutdowns of the units. An enhanced linearization technique was used to solve the non-linearity of the relationship between power and water flow. However, this technique was difficult to apply in practice, given the size of the problem.

Previous research used MILP to facilitate the introduction of the integer variables and the linearization of non-linear functions. In [18] a MILP was developed to maximize the profits by estimating the income of produced power and the start-up/shut-down costs. An analysis of the linearization effects of non-linear functions and related constraints on solution feasibility was conducted. It was found that the linearization may result in solutions that cannot be used in practice, for example, because they do not respect the restricted operational zone constraint. These zones are the periods in which some units cannot operate due to cavitation phenomena, mechanical vibration or loss in efficiency. To obtain a feasible solutions, an approximation of the error caused by MILP approximation formulation was done.

1.3. Contributions

All of the cited works make progress towards solving the problem by using different approximation and linearization techniques. These give good results. Therefore, instead of approximating the hydropower production functions and discretizing the water discharge in order to find the best value that maximizes the energy produced, the efficiency curves of water discharge and power produced for each possible combination of turbines can be used. These curves allow to determine a set of efficiency points of water discharge and it is at these points that the maximum of power produced is reached.

This paper proposes a mathematical formulation of the unit commitment problem where efficiency points of power produced and water discharge for each combination of active turbines are derived and the model selects one of these points to maximize the energy produced. Energy losses caused by the unit start-ups are taken into account. Transmission constraints, the reserve and load constraints are not considered in this paper. The solution provided by the model can directly be implemented in practice since it is obtained on the efficiency points, as the engineers want to operate the power plants. Moreover, using the efficiency points decreases the number of parameters and variables and makes the problem easier to solve.

1.4. Organization of the paper

This paper is organized as follows. Section 2 presents the characteristics of the problem and the mathematical model developed. The results are discussed in Section 3 and concluding remarks are presented in Section 4.

2. Short-term hydro-power scheduling problem

The unit commitment problem is used to determine the optimal production plan up to one week. The purpose is to maximize the energy production and penalize unit start-ups. To maximize energy produced, several factors are considered. This section defines the different factors and describes the short-term problem.

2.1. Power production

The production function depends on the water discharge q in m^3/s, the efficiency of the unit η, the gravitational acceleration G in m/s^2, the density ρ in kg/m^3 and the net water head h_n in m which in turn depends on the total water discharge Q (sum of the water discharge and water spillage) in m^3/s and the volume of the reservoir v in hm^3 [6]. The power output p in W from a hydro generating unit is given by the equation:

$$p(q, h_n) = G \times \eta \times h_n(Q, v) \times q \times \rho \tag{1}$$

The net water head is the difference between the forebay elevation h_f, tailrace elevation h_t and the losses caused by the friction in the penstock h_p. The net water head is calculated by:

$$h_n(Q, v) = h_f(v) - h_t(Q) - h_p(Q, q) \tag{2}$$

2.2. Turbine efficiency

Turbine efficiency is the most important factor in the output of the unit. It represents the capability of the turbine to transform the mechanical power of the water into the most electrical power as possible. The turbine efficiency depends on the water discharge and the net water head. Each turbine has its own efficiency curve and for the same value of water discharge and volume, different turbines produce different quantities of power.

2.3. Combinations of the turbines

The total power output depends on the total water discharge and the number of active turbines. In the operational reality, the active turbines are grouped into combinations. Table 1 shows an example of the possible combinations of a powerhouse that has 4 available turbines. The operator can use up to 4 turbines. In the operation of a powerhouse, a minimum number of active turbines (for example 3 turbines) is required due to the physical constraints of the powerhouse.

Table 1: Combinations of 4 available turbines

3 active turbines	4 active turbines
123 124	1234
134 234	

As explained in Section 2.2, each turbine has its own efficiency. The combination of active turbines has also its own efficiency. Fig 1 shows the power output depending on the water discharge for all possible combinations and this for a given forebay elevation.

As shown in Fig 1 , the power output decreases when the maximum water discharge of the turbine combination has been reached. For example, in the instance where the active turbines are

234 (curve with ’square’ marker) the power output decreases once the maximum water discharge is reached (510 m^3/s). It is useless to increase the quantity of water discharge because it will be spilled, therefore the tailrace elevation will increase and consequently the net water head will be reduced. For this reason the power output is decreased. Fig 2 shows that for the same number of active turbines and the same value of water discharge, there is a small variation in the production which confirms that the power produced differs from one combination to another.

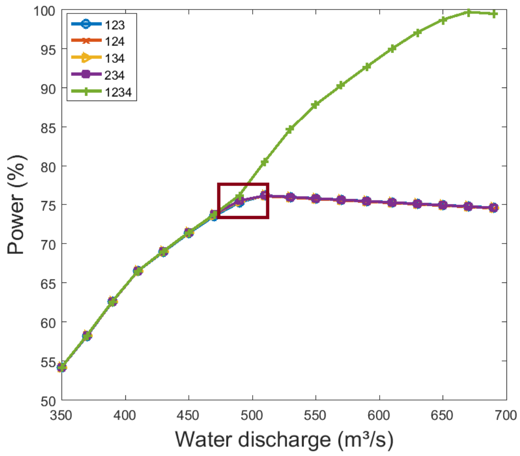

Figure 1: Power output for all combinations

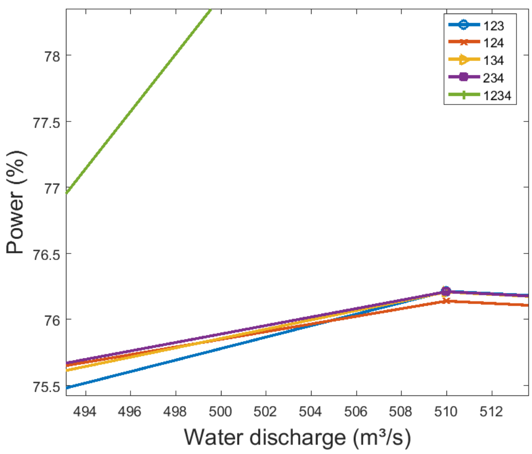

Figure 2: Zoom-in of the rectangle in Fig.1

2.4. Start-ups

The objective of the problem is to determine for each period the best combinations of active turbines in order to maximize the energy produced. The model can select different combinations from one period to another. However, there is an important concept that must be taken into account which is the start-ups of the turbines. Frequent start-ups cause maintenance costs and decrease the life cycle of the turbines. In practice, it is recommended to have a limited number of start-ups.

2.5. Problem description

The objective is to maximize energy production and penalize unit start-ups. Hydropower production function can be modelled in many different ways. Polynomial equations, which determines an analytic expression of a polynomial of degree n passing through data points (p,h_n). The wrong choice of the degree influences the results. Splines allow to split the points into subsets and use a polynomial approximation for each subset of points, then connect them. Other methods can be used such as interpolation, linearization, etc. These methods give good results and allow to find the value of water discharge that maximize the energy produced. But in operational reality, instead of approximating the hydropower production functions and optimizing on all the values of the water discharge to find the best values, the best values are already known.

These values can be determined using the efficiency curves of water discharge for each possible combination. A set of points corresponding to the maximum water discharge and the maximum efficiency of each available combinations group is determined. However, selecting only these points can limit the optimization and lead to infeasible solutions. In this regard, a set of adjacent points to the maximum should be defined. To do so, the convex hull of the efficiency curves for each combinations group is traced. Using the convex hull allows to define the maximum efficiency points. The adjacent points are chosen as $\pm\zeta$ (m^3/s) of the maximum efficiency of water discharge where ζ are integer parameters. The choice of these parameters depends on the management of the powerhouses and the operational precision that can be obtained.

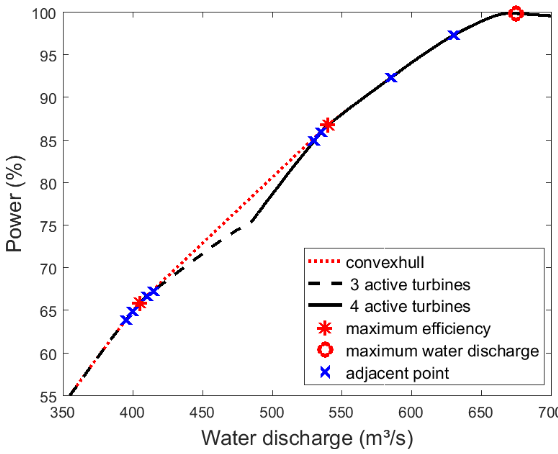

Figure 3: Pairs of efficiency points (P, q)

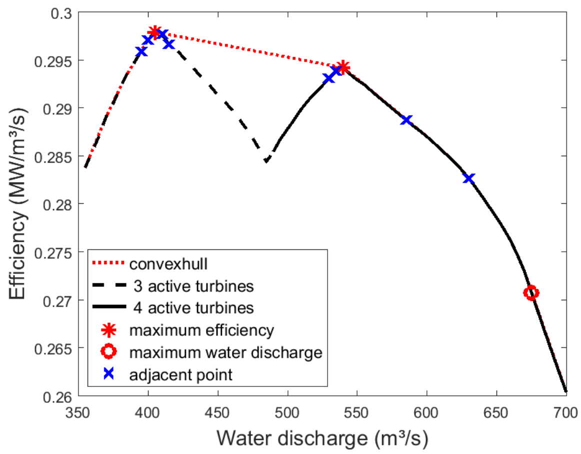

Figure 4: The set of efficiency points of water discharge

For example, Fig.4 shows the efficiency curve of the water discharge (q) for a powerhouse that has 4 available turbines. As explained in section 2.3, the number of active turbines can be 3 (curve with dashed line) or 4 (curve with solid line). For each curve, the maximum efficiency (marked as $'*'$) is defined. The maximum water discharge (marked as $'o'$) is defined only for the curve with the maximum number of turbines because it is the only instance where spillage is an option. The adjacent points (marked as $'X'$) are defined as -3 m^3/s and -6 m^3/s and two equidistant points between the maximum water discharge and the efficiency water discharge. For the curve with a dashed line (3 active turbines) the adjacent points are defined with $\pm$ 3 m^3/s and $\pm$ 6 m^3/s. Once these efficiency points of water discharge are defined, the power produced P associated to the points is determined. Finally the pairs of water discharge and power produced $(P,\, q)$ associated with each efficiency point can be determined for each possible combinaison as shown in Fig.3.

At period t, the best efficiency point of the set is selected respecting some constraints like the water balance to maximize the energy produced. At period $t+1$ another efficiency point can be selected and the combination can be changed. To decrease the number of changes, a penalty factor must be added for each start-up. This penality factor corresponds to the energy loss caused by the the start-up of the unit in (MW).

The set of pairs of efficiency points presented in the previous curves is obtained with a given forebay elevation. The idea is to fix the value of the forebay elevation at the maximum volume of the reservoir V_{max}. Using V_{max} reduces the number of variables and facilitates the resolution of the problem. However, in reality, the reservoir is not always full. The model presented in this paper takes into account this aspect and makes a correction between the produced power at V_{max} and the power obtained at the optimized volume. To do so, a coefficient θ that approximates the power losses due to the volume difference, in proportion to the maximum volume of the reservoir is given by Eq.(3):

$$p(q, h_n) \approx p(q, h_n^{\max}) - \Psi \times (h_n^{\max} - h_n) \tag{3}$$

Where $h_n^{\max}$ is the maximum net water head, hence the volume is equal to the maximum volume of the reservoir ($V_{\max}$). Several curves (p, q) were defined with different volume levels: the minimum, the average, etc. Using Eq.(3), for each curve, a coefficient Ψ corresponding to the solution of the least squares between the true production values and their approximations at V_{max} is determined. The average of all the obtained coefficients gives the value of θ. Since each reservoir has its volume limits, the coefficient θ is calculated for each powerhouse.

2.6. Mathematical model

This section presents the mathematical model used to solve the short-term unit commitment problem. The goal of the short-term hydropower problem is to define the optimal unit commitment for the next days up to one week in order to maximize the energy production and penalize unit start-ups. The problem is formulated as a MILP, that trades off the maximum efficiency of generation versus start-up of units. The formulation is given by a three-terms objective function but remains mono objective since they are not conflicting. The first term computes the power output $P_{k,t}^c$ at each point for each combination at each period. This power is determined at the maximum reservoir volume V_{max}. Since the power output is calculated at V_{max}, the second term makes a correction between the power produced at the current volume and V_{max}. In this regard, the second term of the objective function equals the difference between the volumes, multiplied by an estimate of the energy losses θ^c. The last term of the objective function reduces the number of changes by penalizing unit start-ups. Finally the sum of the three terms is multiplied by Δ_t to convert power to energye, which is maximized. The MILP is given by:

$$max_{y,v,z} \Delta_t \times [\sum_{c\in C}\sum_{t\in T}\sum_{b\in B}\sum_{k\in K_b^c} P_{k,t}^c \times y_{k,t}^c - \sum_{c\in C}\sum_{t\in T} \theta^c \times (V_{max}^c - v_t^c) - \sum_{c\in C}\sum_{t\in T}\sum_{j\in J_t} \varepsilon^c \times z_{j,t}^c] \tag{4}$$

Subject to:

$$v^c_{t+1} = v^c_t - \Delta_t \times [\sum_{b \in B} \sum_{k \in K^c_b} (q^c_{t,k} \times y^c_{k,t} \times \beta) + (\delta^c_t \times \beta) - (d^c_t \times \beta) + \sum_{l \in U^c} \sum_{b \in B} \sum_{k \in k^l_b} (q^l_{t,k} \times \ y^l_{k,t} \times \beta) + (d^l_t \times \beta)] \quad , \forall c \in C, \forall t \in T, \tag{5}$$

$$\sum_{b \in B} \sum_{k \in K^c_b} y^c_{k,t} = 1 \quad , \forall c \in C, \forall t \in T, \tag{6}$$

$$\sum_{b \in B} \sum_{k \in K^c_b} y^c_{k,t+1} \times A^c_{t+1,k,j} - \sum_{b \in B} \sum_{k \in K^c_b} y^c_{k,t} \times A^c_{t,k,j} \leq z^c_{j,t} \quad , \forall c \in C \ , \forall t \in T \ , \forall j \in J_t, \tag{7}$$

$$\sum_{t \in T} \sum_{j \in J_t} z^c_{j,t} \leq N^c_{max} \quad , \forall c \in C, \tag{8}$$

$$v^c_{min} \leq v^c_t \leq V^c_{max} \quad , \forall c \in C, \forall t \in T, \tag{9}$$

$$v^c_1 = v^c_{ini} \quad , \forall c \in C, \tag{10}$$

$$v^c_T \geq v^c_{final} \quad , \forall c \in C, \tag{11}$$

$$y^c_{k,t}, y^l_{k,t}, z^c_{j,t} \in \mathcal{B} \quad , \forall c \in C, \forall t \in T, \forall b \in B^c, \forall k \in K^c_b, \forall l \in U^c \tag{12}$$

$$d^c_t, d^l_t, v^c_t \in \mathcal{R}^+ \quad , \forall c \in C, \forall t \in T, \forall l \in U^c. \tag{13}$$

Constraints (5) ensure the water balance of the powerhouses. In the instance where the powerhouses are in series, the water discharge $q^l_{k,t}$ and the water spillage d^l_t of the upstream powerhouses to the volume v^c_t were added. Otherwise, there will not be taken into account. Constraints (6) force the model to choose only one operating point at each time period for each powerhouse. Constraints (7) are the link between start-up variables and chosen combination considering the set of points. To limit the number of turbine changes during the planning horizon, a maximum number of start-ups N_{max} is imposed with constraints (8). Constraints (9) limit the water in the reservoir of each powerhouse, and (10)-(11) specify initial and final volumes. Finally, (12) defines the binary variables and (13) the real variables.

3. Case study

The MILP model presented in the previous section was tested with data from the Saguenay-Lac-St-Jean hydroelectric system owned by Rio Tinto. Rio Tinto is a company that produces aluminium in the Saguenay region of the province of Quebec in Canada. The production of aluminium requires a lot of electric energy. Rio Tinto has a hydropower system that fills 90% of its energy needs. Rio Tinto has five powerhouses with 44 turbines. Four reservoirs are available with different capacities.

Two powerhouses in the company's system were considered: Chute-Du-Diable (CD) and Chute-Savane (CS). These two powerhouses are in series and each has five turbines. To schedule hydropower production, Rio Tinto aims to use the maximum efficiency of their turbines in order to maximize the energy produced by defining the best combination of turbines operating at each time period. Currently, they use a dynamic programming algorithm and rely on their staff's experience. The selection of the quantity of water discharged and the combination of active turbines are determined by hand. Moreover, the correlation between the powerhouses is not taken into account. The model proposed in this paper is used to define an automatic procedure which allows to determine the best schedule by defining the best combination of active turbines to maximize the energy production. Turbine start-ups are penalized in the objective function to account for the corresponding power losses. The price of the energy is not considered because in the province of Quebec, where this model is tested, the market is regulated by Hydro-Québec, therefore all producers must buy and sell to them and negociate fixed price contracts every year.

3.1. Experimental setup

For both powerhouses, operational restrictions require a minimum of three active turbines. The number of possible combinations is thus 16 for each powerhouse as shown in Table 2: 10 combinations of three turbines, 5 combinations of four turbines and 1 combination of five turbines.

Table 2: Combination of 5 available turbines

3 active turbines	4 active turbines	5 active turbines
123 124	1234	12345
125 134	1235	
135 145	1245	
234 235	1345	
245 345	2345	

For each combination, the pair of points of water discharge and power produced (P, q), as explained in Section 2.1, are defined. The number of adjacent points was defined by making several tests. First, only the maximum point was chosen, but in some cases, this leads to infeasible solutions. Other points were then added. Firstly, two adjacent points were added and feasible solutions were found. To avoid the risk of infeasibility, two other adjacent points were added to assure that the solution is feasible and to allow the model to be more flexible. Since the discretization of the water discharge is done in steps of 5 m^3/s, the adjacent points are defined with $\pm 5\text{m}^3/s$ and $\pm 10\text{m}^3/s$ of water discharge and two equidistant points between the maximum water discharge and the efficient water discharge are also selected (where the number of active turbines is equal to the number of available turbines).

The model was tested on 65 test instances. The duration of each instance is four days partitioned into periods of 1 hour, for a total of 96 hours. The quantity of natural inflows and the availability of the turbines are known and obtained from the historical database. The model is deterministic, therefore there is no uncertainty in the inflows. Initial and final volumes of each reservoir are also provided.

The Xpress [19] solver accessed via Python was used to solve the proposed formulation. The problem was solved to optimality and the MIP gap was equal to 0.01%. The resulting MILP problem has 10560 binary variables, 386 real variables and 2060 constraints. To solve this unit commitment problem, the computational time ranged between 5 and 15 minutes for each instance, on a laptop computer with an Intel Core i5 Processor and 8 GB of RAM.

3.2. Computational results

The results from the proposed model are compared with the real operational decisions taken from the database. Periods with turbines unavailable due to maintenance or repair, with a contingency energy request, and the periods with high water spillage were not selected for comparison. The initial volume, the final volume and the states of turbines at $t = 0$ are equal to those in the database.

Table 3 reports for each instance and for the 96 hours, the energy production optimized by the proposed model, the energy obtained from the real operational decisions, and the average improvement as the difference between these energies divided by the energy obtained from the real operational decisions:

$$\textit{Average improvement(\%)} = \frac{\textit{(energy production optimized (MW) - energy in database (MW))} \times 100}{\textit{energy in database (MW)}}$$

Note that all the results are reported as percentages because the actual values are confidential. To do so, all energies are divided by the maximum value of the energy produced for the 65 instances and multiplied by 100:

$$\textit{Percentage(\%)} = \frac{\textit{energy production (MW)} \times 100}{\textit{max (energy production optimized (MW), energy in database (MW))}}$$

A positive improvement indicates that the optimized solution is better than the real operational decision in the database.

Table 3: Total energy production and average improvements

Instance	Optimized (as a %)	Operational decision (as a %)	Improvement (%)	Instance	Optimized (as a %)	Operational decision (as a %)	Improvement (%)
1	96,00	95,71	0,31	34	81,54	80,20	1,64
2	92,74	91,56	1,27	35	82,36	81,64	0,87
3	74,70	74,19	0,68	36	78,25	77,43	1,05
4	80,92	80,14	0,95	37	84,07	83,60	0,57
5	72,28	71,99	0,40	38	82,81	81,99	0,99
6	79,24	78,76	0,61	39	61,09	60,47	1,01
7	71,95	71,70	0,35	40	69,48	68,77	1,01
8	71,64	71,03	0,85	41	70,25	70,05	0,28
9	87,51	87,00	0,59	42	72,68	72,17	0,71
10	96,23	95,52	0,74	43	71,14	71,09	0,07
11	88,04	87,09	1,08	44	74,63	74,39	0,33
12	80,63	80,03	0,74	45	75,62	75,37	0,34
13	85,44	84,38	1,24	46	77,29	76,95	0,43
14	75,60	75,13	0,62	47	77,53	77,22	0,40
15	75,64	75,29	0,47	48	78,31	77,93	0,49
16	74,43	74,05	0,50	49	77,29	76,57	0,94
17	79,64	79,01	0,79	50	77,19	76,59	0,78
18	72,45	72,01	0,61	51	100,00	99,27	0,73
19	69,25	69,21	0,06	52	81,34	80,59	0,92
20	72,34	72,08	0,36	53	88,74	88,51	0,26
21	72,82	72,00	1,13	54	84,99	84,17	0,97
22	56,39	55,91	0,85	55	97,89	96,73	1,18
23	95,99	95,43	0,59	56	93,25	92,58	0,72
24	94,95	93,94	1,06	57	88,92	87,30	1,82
25	96,49	95,84	0,67	58	81,33	79,92	1,74
26	90,59	89,75	0,92	59	82,88	82,37	0,61
27	90,24	89,66	0,65	60	83,66	83,35	0,37
28	88,05	87,38	0,77	61	79,31	78,56	0,94
29	83,66	82,79	1,04	62	80,21	79,36	1,06
30	62,75	62,34	0,66	63	80,51	79,77	0,92
31	80,30	78,57	2,16	64	82,37	82,02	0,42
32	83,22	80,85	2,85	65	73,33	72,78	0,75
33	88.24	87.30	1.07				

Fig 5 is a histogram comparing the average improvements of energy production between the optimized solutions and the real operational decisions. Fig 5 shows that 32 instances have an average improvement between 0.5% and 1% and 17 instances have an average improvement greater than 1%. Given the current prices of energy in the province of Quebec, an increase of 1% in the energy production means approximately \$14000 of savings in just 4 days (96 hours). Therefore, on an annual basis and considering that this study considers only half of the power plants of the company, the potential savings are substantial.

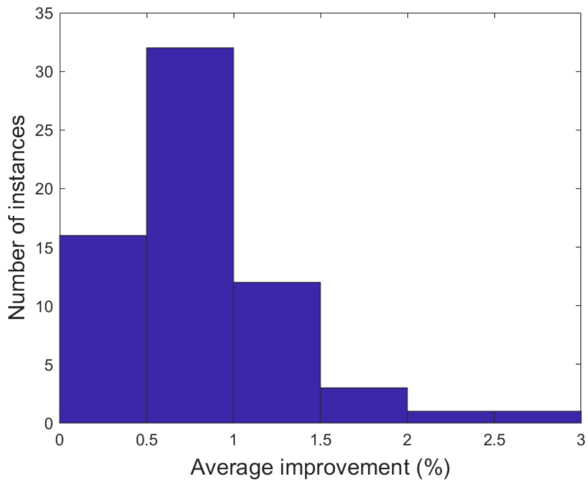

Figure 5: Histogram of average improvements of energy

3.3. Validation of the model

To validate the values of the objective function, the energy estimated by our model was compared with the real value of the production function at Rio Tinto for the same volume of the reservoir, the same water discharge and the same combination of active turbines. Table 4 shows the difference between the energy approximated and the real values at Rio Tinto for some instances. The results show that the values are very close, which confirms the accuracy of our model.

Table 4: Comparison between the energy approximated and the real values

instance	difference (%)
31	0.01
32	0.01
36	0.07
65	0.03

3.4. Interpretation of the results

Let us analyze instance 32 where the average improvement reaches 2.85% (as shown in Table 3). Fig 6 and 7 show that for the same initial and final volumes, the optimized solutions (solid line) and the the real operational decisions (dashed line) are different. For CD, the real operational decision requires the activation of 4 turbines during the whole planning horizon. The optimal solution requires the activation of 4 turbines during the first 27 periods until the volume reaches the maximum value, then the activation of 5 turbines for the next periods. The estimated power loss caused by the start-up of the fifth turbine is negligible compared to the expected power gained when operating with 5 turbines. For CS, the number of active turbines of the optimized and the real operational decisions are equal. However, there is a difference between the produced power of the optimized solution and the real operational decision. This difference is due to the suitable choice of the quantity of water discharge. In the optimized solution, the value of the water discharge is selected according to the efficiency points. For this reason, the quantity of produced power of the optimized solution is more than that of the real operational decision.

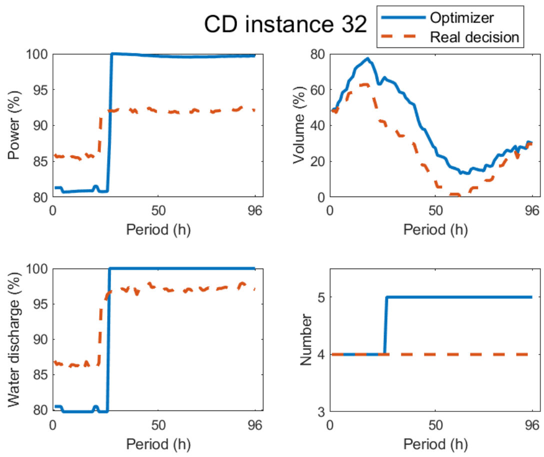

Figure 6: instance 32 for CD

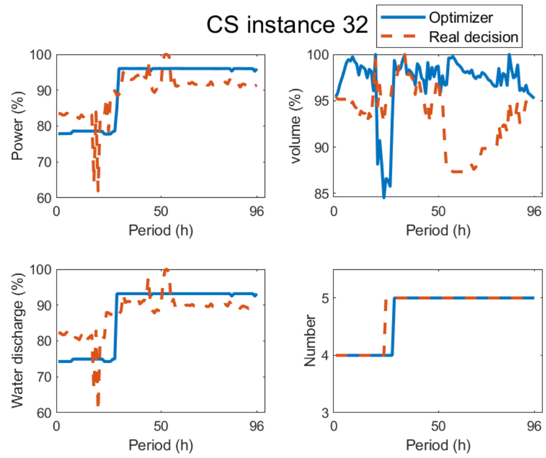

Figure 7: instance 32 for CS

For instances 36 and 22, the average improvement is respectively 1.05% and 0.85%. Fig 8 and 10 show that for CD in both instances, the number of active turbines and the volume of the reservoir of the optimized solution and the real operational decision are similar. Since the optimized solution uses the efficiency points of water discharge, the produced power is higher than the real operational decision.

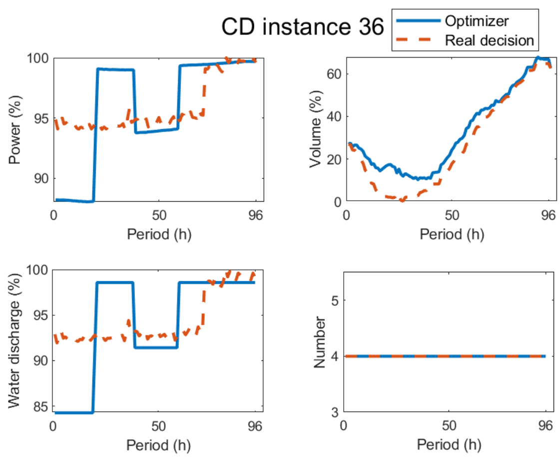

Figure 8: instance 36 for CD

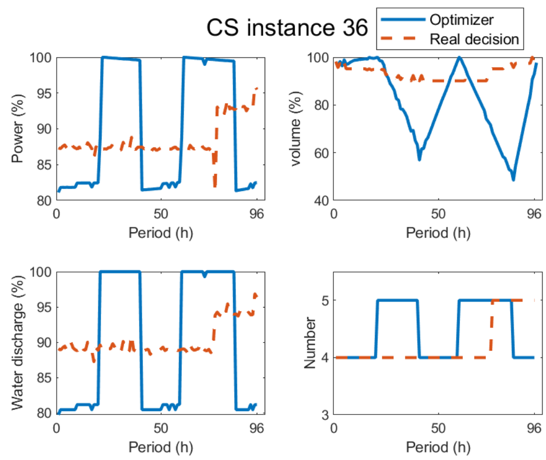

Figure 9: instance 36 for CS

An analysis of the results for CS shows that the total power output of the optimized solution is higher than the power output of the real operational decision in both instances. Fig 9 and 11

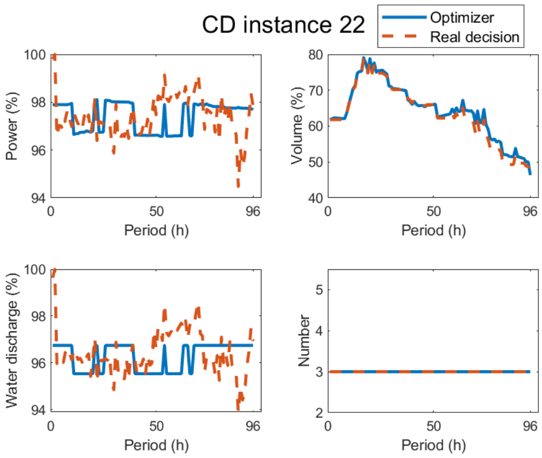

Figure 10: instance 22 for CD

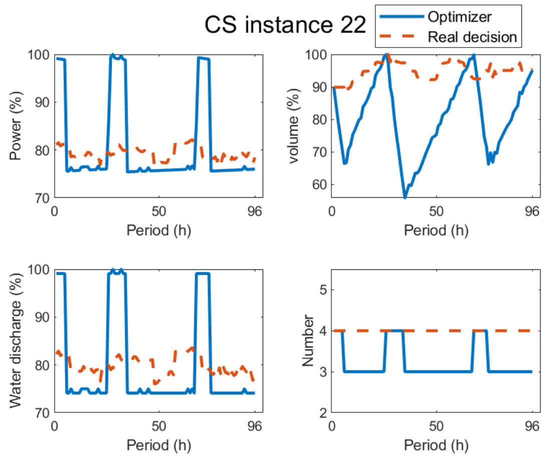

Figure 11: instance 22 for CS

show that in the optimized solution, the number of the active turbines switches between two values. For example, for instance 36 (Fig 9), the optimized solution requires the activation of 4 turbines during the first 20 periods to reach the maximum volume of the reservoir. To decrease the volume, the number of active turbines is changed from 4 to 5 for the following 19 hours. At period 20, the number of active turbines is changed again to increase the volume. Notice that there is a cycle trend for these two instances. Future research will focus on this phenomenon.

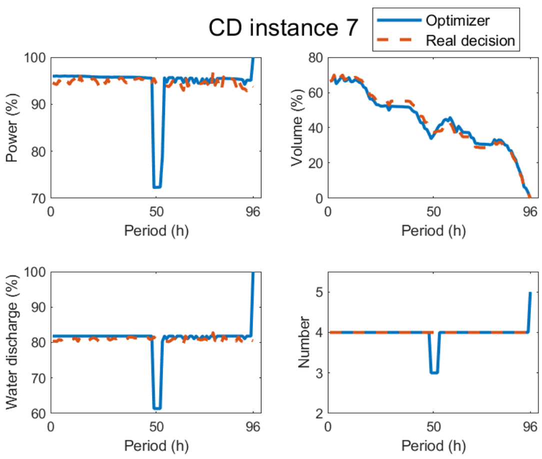

Figure 12: instance 7 for CD

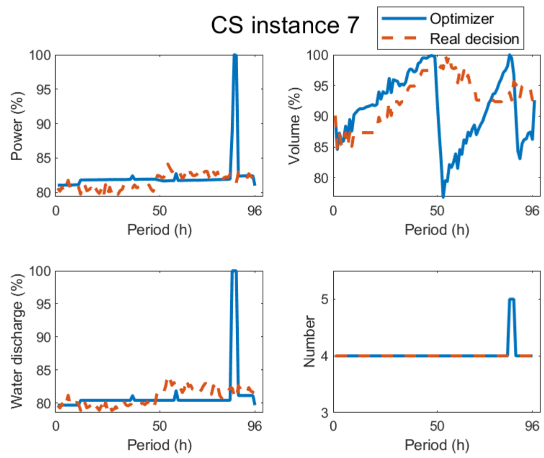

Figure 13: instance 7 for CS

In instance 7, the average improvement is 0.35%. Fig 12 and 13 show that the number of active turbines, the quantity of water discharge and the produced power of the optimized and the real operational decisions are even similar. Nevertheless, there is a change in the curves at period 49 for CD and at period 85 for CS. At these periods, there is a descent in the volume of the CS. This is due to the decline of the quantity of the natural inflows in the reservoir. Since the inflows are deterministic and taken from the operational database, there is likely a measurement error in these periods. For this reason, future research will look at implementing a method to generate the inflows and develop a stochastic model.

3.5. Discussions

The obtained results for the 65 instances show that the proposed model allows to produce more energy than the energy obtained from the real operational decisions. The accuracy of the model is confirmed since the results are compared to the real value of the production function at Rio Tinto. Therefore, the optimized solutions are always on the efficiency points, as the engineers want to operate the powerhouses. The advantage of this model is that the solution can be directly implemented in practice.

4. Conclusion

This paper introduced an innovative method to optimize the unit commitment of hydropower generating units. This method allows the producer to implement directly the optimized solution since it reflects the operational reality. The novelty is to determine the pair of points of maximum efficiency for water discharge and the power produced at the maximum storage, then operate the turbines at these points. The points are determined for all possible combinations of turbines. Since the reservoir is not always full, a correction of produced power was done. To avoid start-ups, a maximum number of turbine changes is imposed to find a viable solution in practice. The problem is formulated using a MILP to find the exact combination of turbines and the best operation point that maximizes total energy production while penalizing start-ups. The model was tested using data for two powerhouses from a real world system and the optimal operation schedules were successfully obtained. The solution found by solving the optimization problem can be directly implemented in practice. Future research will consider increasing the number of powerhouses in order to solve a more realistic and challenging system, and developing a stochastic model that takes into account the uncertainty of inflows and study the effects of the cycles.

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