

Modeling Tropical Cyclone Boundary Layer: Height-Resolving Pressure and Wind Fields

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Abstract: The high-accurate wind field of a tropical cyclone boundary layer, which is essentially governed by the Navier-Stokes equations, could be efficiently obtained by predefining the pressure field. Conventionally, the prescribed pressure field is a 1-D function varying with the distance to the cyclone center (radius). In this study, the pressure field model has been extended to a 2-D function with respect to both radius and height. In addition, a number of field measurements in the tropical cyclone boundary layer indicate rapid variation of the thermodynamic temperature and moisture with time and space. Hence, their effects on the wind field were considered in terms of the virtual temperature, which was integrated in the modeling of pressure field. The analytical solutions of the wind field, as a sum of gradient and frictional wind components, were derived based on a height-resolving scheme using the updated pressure field. Since the tropical cyclone gradient wind and depth of boundary layer are mutually dependent, the iteration approach was utilized in the computation. The proposed height-resolving pressure and wind analytical models have been comprehensively validated with the global positioning system (GPS)-based dropsonde data. The significant importance to consider the height-varying pressure, thermodynamic temperature and moisture in the modeling of the wind field in the tropical cyclone boundary layer were also demonstrated.

Keywords: *Tropical cyclone, Height-resolving model, Boundary layer wind, Pressure, Temperature, Moisture.*

25 **1. Introduction**

26 The wind field in the boundary layer region of a mature tropical cyclone is of great significance
27 since a substantial part of economic and life losses result from the events directly or indirectly
28 related to high winds, e.g., wind damage to structures, wind-driven storm surge and wind-rainfall
29 interaction. The situation has become more challenging due to the changing climate and continued
30 escalation of coastal population. While there have been considerable advances in improving the
31 simulation accuracy of tropical cyclone wind field based on the numerical weather prediction
32 models associated with a significant increase of observation data, they are not practical to be
33 employed in the assessments of risk posted by wind-related hazards due to their high
34 computational demands. The state-of-the-art wind hazard risk analysis is essentially based on the
35 Monte Carlo methodology proposed by Russel (1971), where a large number of scenarios need to
36 be carried out. In this context, the parametric, engineering models for tropical cyclone wind fields,
37 based on the prescribed pressure fields, have been popularly utilized.

38 While several studies have shown that the height-resolving models are superior to the slab
39 (depth-averaged) models that treat the boundary-layer height of the tropical cyclone as a constant,
40 both of these two high-efficient wind field simulation schemes are widely employed in engineering
41 applications. Although the hydrostatic equation simply indicates the pressure field depends on the
42 height, both the slab and height-resolving models conventionally assume the prescribed pressure
43 field is unchanged through the depth of the boundary layer. In particular, the 1-D empirical model
44 introduced by Holland (1980) for pressure, varying with the distance to the cyclone center (radius),
45 has been extensively used due to its simplicity and consistency with field measurements (e.g., Zhao
46 et al. 2013; Mudd et al. 2014). Recently, Huang et al. (2012) integrated the effects of temperature
47 and variation of central pressure difference with height into the prescribed pressure field for more

48 accurately simulating the typhoon wind field using Meng's model (Meng et al. 1995). Since the
49 gradient wind speed in this refined Meng's model varies with the height from the ground, it is not
50 easy to select the appropriate value in the calculation.

51 Following the pioneering work of Huang et al. (2012), the 1-D Holland's empirical
52 pressure model has been extended to a 2-D function with respect to both radius and height in this
53 study. Since a number of field measurements in the tropical cyclone boundary layer indicate rapid
54 variation of the thermodynamic temperature and moisture with time and space, their effects on the
55 wind field were considered in terms of the virtual temperature, which was integrated in the
56 modeling of pressure field. The obtained 2-D formula for pressure $p(z, r)$ explicitly includes the
57 temperature lapse rate parameter Γ . The global positioning system (GPS)-based dropsonde data
58 (e.g., temperature, humidity, pressure) for the tropical cyclones further demonstrated a heavy
59 dependence of Γ and hence pressure on the moisture content. Furthermore, the proposed 2-D
60 pressure formula indicates the consideration of climate changes (e.g., global warming) may have
61 significant implications to the wind field simulation of a tropical cyclone. The analytical solutions
62 of the wind field were derived based on a recently developed height-resolving scheme (Snaiki and
63 Wu 2016) using the obtained 2-D pressure field. To select an appropriate height for the calculation
64 of gradient wind speed, the iteration approach was utilized using the depth scale of the tropical
65 cyclone boundary layer as the initial value. The proposed height-resolving pressure and wind
66 analytical models have been comprehensively validated with the GPS-based dropsonde data. The
67 significant importance to consider the height-varying pressure, thermodynamic temperature and
68 moisture in the modeling of the wind field in the tropical cyclone boundary layer were also
69 demonstrated.

70

71 2. Height-resolving pressure field

72 In the simulation of the wind field inside the boundary layer of a tropical cyclone, the surface level
73 pressure profile is typically prescribed to efficiently solve the horizontal momentum equations. In
74 general, the atmospheric pressure can be expressed by the state equation for ideal gas as follows:

$$75 \quad p = \rho RT \quad (1)$$

76 where ρ = air density; R = ideal gas constant; and T = temperature.

77 2.1 Moisture effects

78 The warm, moist air is considered as the fuel of the tropical cyclones. To simultaneously account
79 for the temperature and moisture effects, a convenient way to proceed would be the use of the
80 virtual temperature T_v which is expressed as follows:

$$81 \quad T_v = T \left[\frac{R_{noa} + r_v R_v}{R_{noa} (1 + r_v)} \right] \quad (2)$$

82 where r_v = mixing ratio of water vapor; R_v = gas constant of the water vapor; and

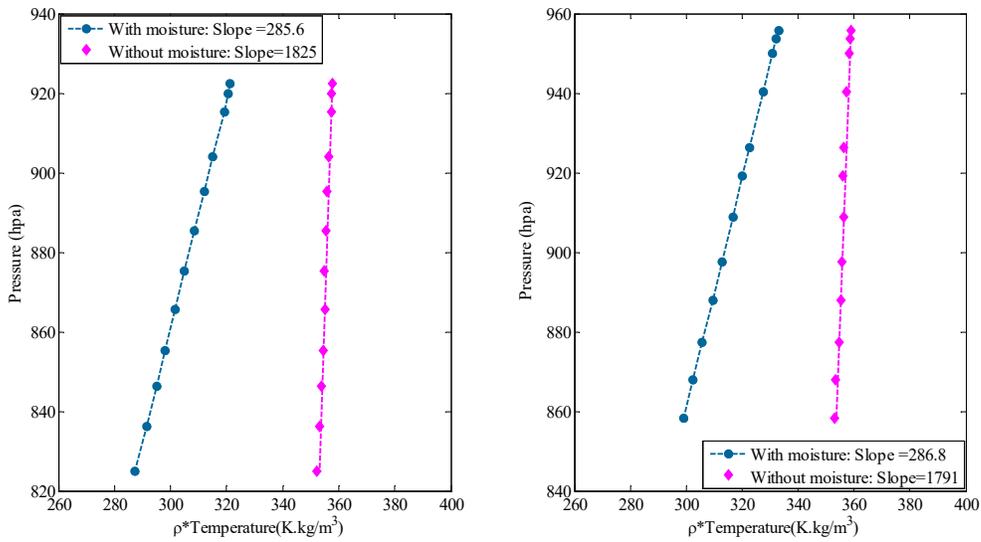
83 $R_{noa} \approx 287 \text{ J kg}^{-1} \text{ K}^{-1}$ is the gas constant of mixture of nitrogen (N_2), oxygen (O_2) and argon (Ar).

84 R_{noa} will be denoted subsequently by R for convenience. Accordingly, the state equation can be
85 extended to include the virtual temperature:

$$86 \quad p = \rho RT_v \quad (3)$$

87 The importance of moisture consideration in the pressure simulation for a typical tropical
88 cyclone will be illustrated through two dropsonde measurements collected by the National
89 Hurricane Center and Hurricane Research Division during hurricane Katrina. The dropsonde IDs
90 are (051926111) and (051926170), respectively. The dropsondes are usually launched from an

91 altitude of 3 km or higher and provide high-resolution thermodynamics data, namely temperature,
 92 pressure, and humidity. To ensure quality control, collected data are post-processed. Figure 1
 93 presents the pressure p as a function of $\rho_{humid\ air} * T_v$ and of $\rho_{dry\ air} * T$, respectively. It is shown the
 94 consideration of moisture gives a slope of 285.6 that is close to the gas constant $R \approx 287\ J\ kg^{-1}\ K^{-1}$
 95 , while the dry assumption results in a slope much larger than this value. This indicates the
 96 importance of moisture to accurately simulate the pressure field in the tropical cyclone.



97
 98 **Fig. 1** Moisture effects on the state equation based on two dropsonde measurements during hurricane Katrina (Left:
 99 051926170 and Right: 051926111)

100 2.2 Pressure formula

101 To derive the pressure expression, the state equation is first applied on the surface level which
 102 gives:

$$103 \quad p_0 = \rho_0 R T_{v0} \quad (4)$$

104 where ρ_0 = surface air density; and T_{v0} = surface virtual temperature.

105 Combining the Eqs. (3) and (4) yields the following expression:

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$$\frac{p}{p_0} = \frac{\rho T_v}{\rho_0 T_{v0}} \quad (5)$$

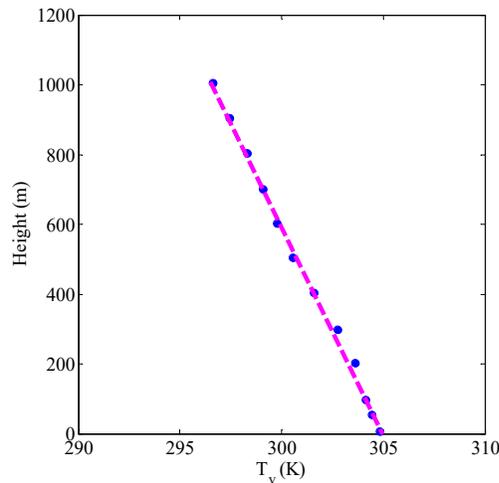
107 The surface pressure is given based on the widely-used Holland's formula:

108
$$p_0 = p_{c0} + \Delta p_0 \cdot \exp\left[-\left(r_m / r\right)^B\right] \quad (6)$$

109 where p_{c0} = central pressure at the surface; Δp_0 = central pressure difference at the surface; r_m =
 110 radius of maximum winds; r = radial distance from the tropical cyclone center; and B = Holland's
 111 radial pressure parameter. Hence the pressure can be expressed as:

112
$$p = \frac{\rho T_v}{\rho_0 T_{v0}} \left(p_{c0} + \Delta p_0 \cdot \exp\left[-\left(r_m / r\right)^B\right] \right) \quad (7)$$

113 On the other hand, it is well known that temperature of air decreases with height. More
 114 specifically, it is approximately a linear function of height for relatively low altitudes, as shown in
 115 Fig. 2. The data in Fig. 2 is provided by the dropsonde (01074007) during hurricane Katrina.



116
 117 **Fig. 2.** Temperature profile of hurricane Katrina corresponding to the dropsonde ID (01074007)

118 Accordingly, the temperature could be approximated as:

119
$$T_v = -\Gamma z + T_{v0} \quad (8)$$

120 where z = vertical coordinate; and Γ = temperature lapse rate. Therefore, the pressure formula

121 becomes:

$$122 \quad p = \frac{\rho}{\rho_0} \left(-\frac{\Gamma}{T_{v0}} z + 1 \right) \left(p_{c0} + \Delta p_0 \cdot \exp \left[-\left(r_m / r \right)^B \right] \right) \quad (9)$$

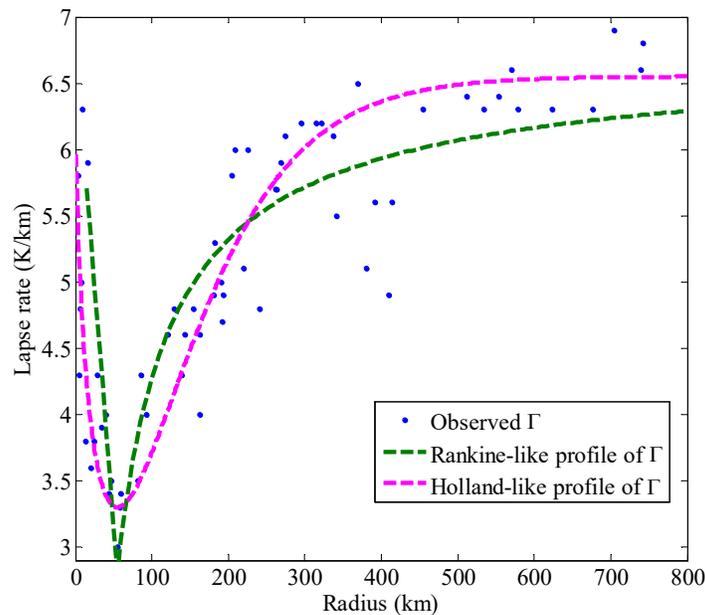
123 If the moisture was disregarded, one could determine the value of the so-called dry
 124 adiabatic lapse rate for dry simulation according to the first law of thermodynamics

$$125 \quad dq = c_p dT - \frac{1}{\rho} dp, \text{ where } c_p = \text{specific heat capacity of air; } dq = \text{heat transfer} = 0; \text{ and } dp = -\rho g dz$$

126 based on the hydrostatic equation. As a result, the dry adiabatic lapse rate can be obtained as
 127 follows:

$$128 \quad \Gamma_d = -\frac{dT}{dz} = \frac{g}{c_p} \approx 9.8 \text{ K/Km} \quad (10)$$

129 For a typical tropical cyclone, however, it is noted that the temperature lapse rate changes with
 130 space. Hence, constant Γ_d cannot be adopted for the pressure simulation. Figure 3 presents the
 131 radial variation of the lapse rate for hurricane Gustav (2008) based on 62 dropsondes data.



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Fig. 3. Radial variation of the observed lapse rate for hurricane Gustav (2008)

134 As shown in Fig. 3, the lapse rate varies considerably with the distance from the center of the
135 tropical cyclone. The variation is reasonable since the moisture causes the lapse rate to be smaller
136 (CCSP 2006; IPCC 2000; 2007; 2012). Actually, the lapse rate parameter Γ is an important
137 parameter in the meteorology science to consider the negative feedback from global warming and
138 hence increased moisture (IPCC 2000; 2007; 2012). In the eyewall region where the moisture
139 content is high, there is a rapid drop in the lapse rate (around $3 K / km$). Then the moisture content
140 tends to decrease far away from the eye wall, which leads to an increase of the lapse rate reaching
141 a value around $6.5 K / km$. Similarly, a reduced moisture content results in increase of the lapse
142 rate in the eye region compared to the eye wall region. There is a sudden decrease of the lapse rate
143 in some regions far away from the tropical cyclone center, as presented in Fig. 3 (around
144 $r \approx 400 km$). This is probably because these specific dropsondes were launched in a rainband
145 region where the moisture content was locally increased. It should be noted that only the radial
146 variation of the lapse rate is illustrated in Fig. 3, while, in general, it is also dependent on the
147 azimuthal coordinate. The temperature lapse rate in the tropical cyclone is typically smaller than
148 the dry adiabatic lapse rate $\Gamma_d \approx 9.8^\circ K / Km$.

149 Based on the measured data shown in Fig. 3, two empirical radial profiles of Γ are
150 proposed in this study for convenient applications to the pressure simulations in the tropical
151 cyclone. The first formula is a modified version of Rankine-like profile, which leads to the
152 following expression for the lapse rate:

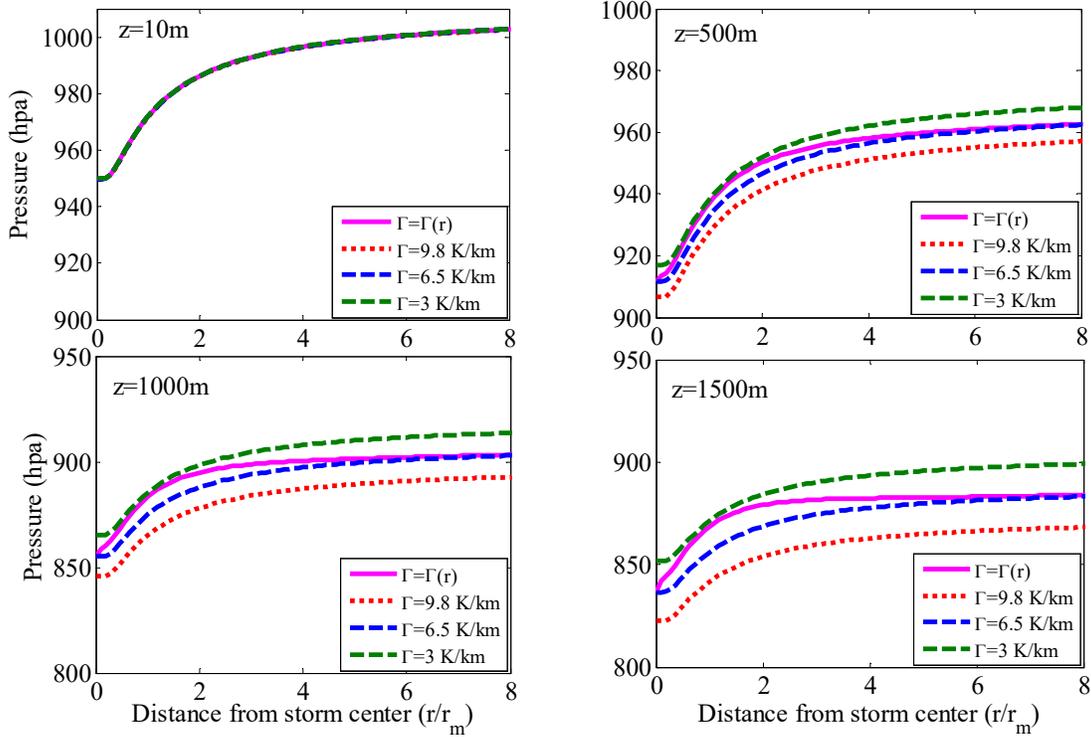
$$153 \quad \Gamma(r) = \begin{cases} (\Gamma_{r_m} - \Gamma_{eye}) \frac{r}{r_m} + \Gamma_{eye} & ; r < r_m \\ (\Gamma_{r_m} - \Gamma_0) \left(\frac{r_m}{r} \right)^a + \Gamma_0 & ; r \geq r_m \end{cases} \quad (11)$$

154 where Γ_{r_m} = lapse rate at the radius of maximum winds; Γ_{eye} = lapse rate at the tropical cyclone
 155 center; Γ_0 = lapse rate in the far field; a = scaling parameter that adjusts the profile shape. The
 156 second formula can be obtained using a modified version of Holland-like profile, which leads to
 157 the following expression:

$$158 \quad \Gamma(r) = (\Gamma_{r_m} - \Gamma_0) \left\{ \left(\frac{r_m}{r} \right)^b \exp \left[1 - \left(\frac{r_m}{r} \right)^b \right] \right\}^{0.5} + \Gamma_0 \quad (12)$$

159 where b = scaling parameter that adjusts the profile shape; Γ_{eye} is assumed to be approximately
 160 equal to Γ_0 for simplification. In general, Eq. (12) provides a smoother profile than that generated
 161 by Eq. (11). The measured data of hurricane Gustav (2008) were fitted using the abovementioned
 162 empirical profiles [i.e., Eq. (11) and Eq. (12)] with parameters of $\Gamma_{r_m} = 2.8K / km$,
 163 $\Gamma_0 \approx \Gamma_{eye} \approx 6.8K / km$ and $a = 0.77$ for the Rankine-like profile and $\Gamma_{r_m} = 3.3K / km$, $\Gamma_0 = 6.55K / km$
 164 and $b = 1.11$ for the Holland-like profile. The fitted profiles are shown in Fig. 3, and the one based
 165 on the Holland-like formula presents a better result according to least squares. It should be stressed
 166 out that the proposed empirical profiles of the lapse rate can be further improved with more data
 167 from the dropsondes.

168 Figures 4 and 5 present, respectively, the radial and vertical pressure profiles based on the
 169 proposed 2-D model [Eq. (9)] with various considerations of the lapse rates. The radial variation
 170 of the temperature lapse rates is obtained using the Holland-like profile. The other parameters are
 171 as follows: $p_{c0} = 950 hpa$; $r_m = 55 km$; $B = 1$; and $T_{v0} = 302 K$. It is shown that the effects of the
 172 changing temperature lapse rate cannot be ignored to accurately simulate the pressure profiles in
 173 the tropical cyclones.



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Fig. 4. Radial pressure profiles for several lapse rates at different altitudes

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As shown in Fig. 4, the correlation of the radial pressure profile with the lapse rate increases with

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the height. In the eyewall region, the proposed pressure profile $[\Gamma = \Gamma(r)]$ is almost identical to

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the pressure profile corresponding to $\Gamma = 3 K / km$, while it coincides with the pressure profile

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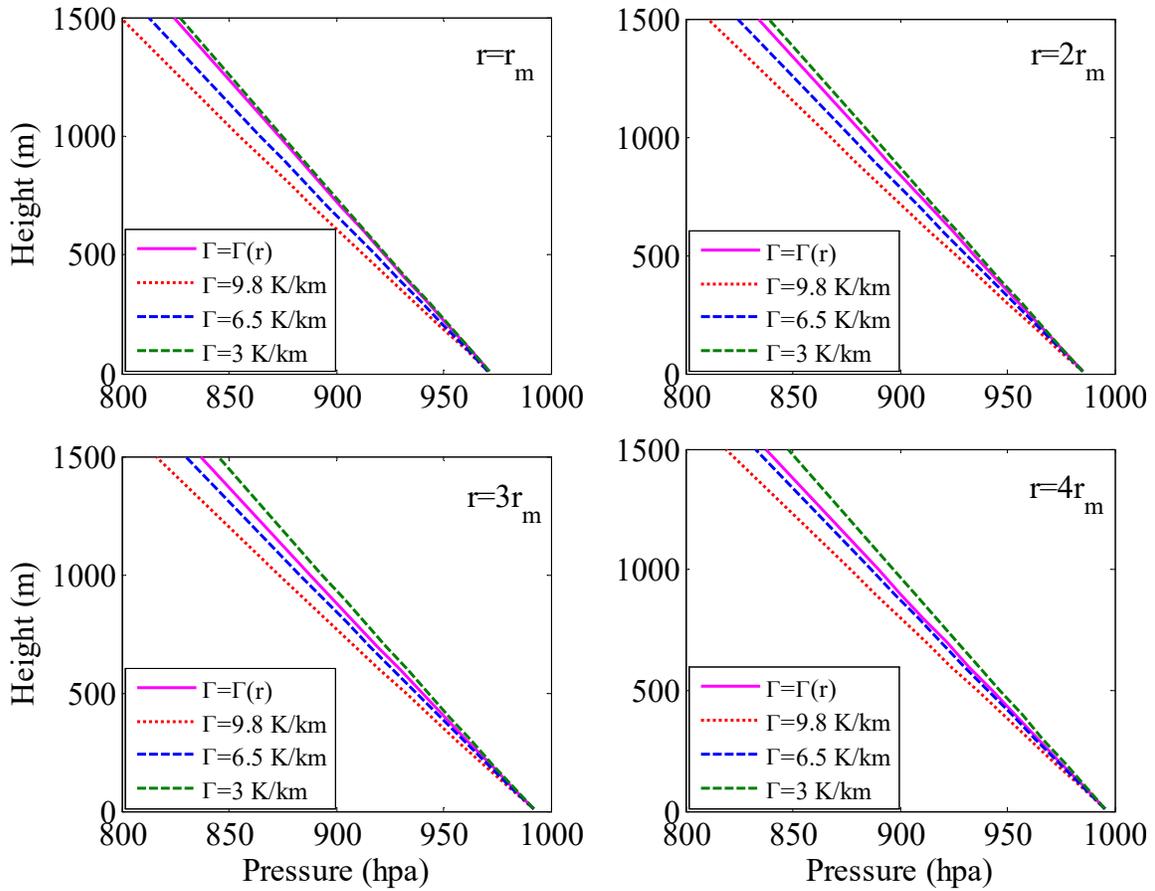
corresponding to $\Gamma = 6.5 K / km$ far away from the tropical cyclone center. This observation is

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further demonstrated by the vertical pressure profiles as presented in Fig. 5. In addition, Fig. 5 also

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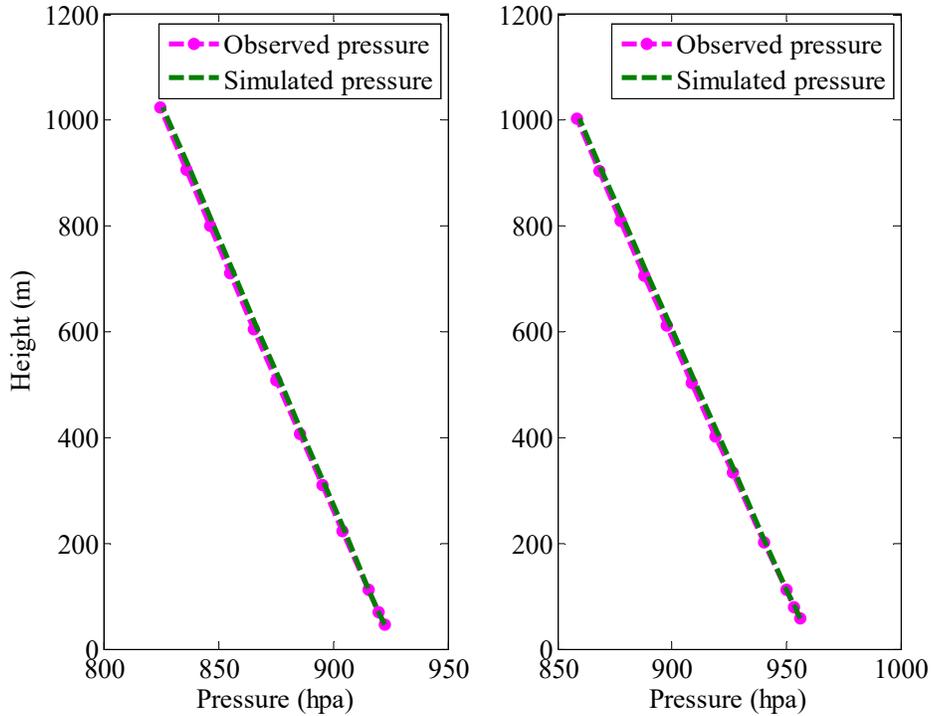
clearly shows that larger differences of the pressure values are obtained at higher altitudes.



182

183 **Fig. 5.** Vertical Pressure profiles corresponding to several lapse rates at different locations inside the tropical
 184 cyclone

185 Using the same data as in Fig. 1, the vertical pressure profile was simulated using the proposed
 186 pressure profile [Eq. (9)]. It could be concluded that both the observed and simulated pressures are
 187 in good agreement as illustrated in Fig. 6.



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Fig. 6. Comparison between the simulated and observed pressures corresponding to two dropsondes during hurricane Katrina (Left: 051926170 and Right: 051926111)

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It is interesting to notice that the developed pressure formula of Eq. (9) may offer an improved method to assess the climate change impacts on the wind field of tropical cyclones. More specifically, the climate change assessment could be considered in terms of the surface central pressure and the vertical pressure profile. Based on the surface central pressure, the future projections of the sea surface temperatures (SST) can be incorporated into the wind field simulations using the relative intensity developed by Darling (1991). Recently, Mudd et al. (2014) carried out simulations to quantify the climate change impact on the northeast US coastal region with this methodology. As pointed out by Mudd et al. (2015), however, considering only the SST will not give accurate results for climate change estimation. Several other environmental parameters that can contribute significantly to the tropical cyclone intensity and are expected to change with global warming, should be accounted for (e.g., air moisture content, temperature at

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202 higher altitudes). For example, the moisture content is believed to increase significantly with
203 global warming (IPCC 2000; 2007; 2012). This indicates reduced lapse rates (Frierson 2005),
204 which could be conveniently integrated into the wind field simulations with the proposed 2-D
205 pressure model.

206 **3. Height-resolving wind field**

207 A linear height-resolving wind field model recently developed by Snaiki and Wu (2016) will be
208 used in this study for wind field simulation. The model not only considers the radial variation of
209 the depth scale of the boundary layer but also accounts for the azimuthal dependence of wind field,
210 resulting an enhanced simulation of the real behavior of a moving tropical cyclone (Snaiki and Wu
211 2016). In this section, a brief discussion of the employed wind field model will first be presented
212 for completeness, followed with the improved wind field simulation by integrating the 2-D
213 pressure field.

214 The governing equation of the wind field could be described as follows:

$$215 \quad \frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v} = -\frac{1}{\rho} \nabla p - f \mathbf{k} \times \mathbf{v} + \mathbf{F} \quad (13)$$

216 where \mathbf{v} = wind velocity; f = Coriolis parameter; \mathbf{k} = unit vector in the vertical direction; and \mathbf{F} =
217 frictional force. In order to solve Eq. (13), the decomposition method is used in which the wind
218 velocity (\mathbf{v}) is expressed as:

$$219 \quad \mathbf{v} = \mathbf{v}_g + \mathbf{v}' \quad (14)$$

220 where \mathbf{v}_g = gradient wind in the free atmosphere; and \mathbf{v}' = frictional component near the ground
221 surface. Therefore two equations can be derived from Eq. (13):

222
$$\frac{\partial \mathbf{v}_g}{\partial t} + \mathbf{v}_g \cdot \nabla \mathbf{v}_g = -\frac{1}{\rho} \nabla p - f \mathbf{k} \times \mathbf{v}_g \quad (15a)$$

223
$$\frac{\partial \mathbf{v}'}{\partial t} + \mathbf{v}' \cdot \nabla \mathbf{v}' + \mathbf{v}' \cdot \nabla \mathbf{v}_g + \mathbf{v}_g \cdot \nabla \mathbf{v}' = -f \mathbf{k} \times \mathbf{v}' + F \quad (15b)$$

224 Similar to Meng et al. (1995), the gradient wind pattern \mathbf{v}_g is assumed to move at the translation
 225 velocity of the tropical cyclone c in the free atmosphere, thus the unsteady term can be expressed

226 as: $\frac{\partial \mathbf{v}_g}{\partial t} = -c \cdot \nabla \mathbf{v}_g$. On the other hand, the unsteady term $\frac{\partial \mathbf{v}'}{\partial t}$ in the tropical cyclone boundary layer

227 is usually considered significantly smaller than the turbulent viscosity and inertia terms, and hence
 228 neglected.

229 3.1 Gradient wind speed

230 Equation (15a) could be solved straightforwardly in the cylindrical coordinate system (Georgiou
 231 1985; Meng et al. 1995). Hence:

232
$$v_{\theta g} = \frac{(-c \sin(\theta - \nu) - fr)}{2} + \left[\frac{(-c \sin(\theta - \nu) - fr)^2}{4} + \frac{r}{\rho} \frac{\partial p}{\partial r} \right]^{1/2} \quad (16)$$

233 where $\nu =$ approach angle (counter clockwise positive from the East). The radial velocity is then

234 obtained from the continuity equation: $v_{rg} = -\frac{1}{r} \int_0^r \frac{\partial v_{\theta g}}{\partial \theta} dr$ which is usually disregarded as suggested

235 by Meng et al. (1995) due to its insignificant effects.

236 3.2 Frictional wind speed

237 In the cylindrical coordinate, Eq. (15b) becomes:

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$$u' \frac{\partial u'}{\partial r} + \frac{v_{\theta g} + v'}{r} \frac{\partial u'}{\partial \theta} + w' \frac{\partial u'}{\partial z} - \frac{v'^2}{r} - \xi_g v' = K \left[\nabla^2 u - \frac{1}{r^2} \left(u + 2 \frac{\partial v}{\partial \theta} \right) \right] \quad (17a)$$

$$239 \quad u' \frac{\partial v'}{\partial r} + \frac{v_{\theta g} + v'}{r} \frac{\partial v'}{\partial \theta} + w \frac{\partial v'}{\partial z} + \frac{u'v'}{r} + \xi_{ag} u' + \frac{v'}{r} \frac{\partial v_{\theta g}}{\partial \theta} = K \left[\nabla^2 v - \frac{1}{r^2} \left(v - 2 \frac{\partial u}{\partial \theta} \right) \right] \quad (17b)$$

240 where (u, v, w) = velocity vector; u', v' = frictional components of the wind velocity; $\xi_g = \frac{2v_{\theta g}}{r} + f$ is

241 the absolute angular velocity; $\xi_{ag} = \frac{\partial v_{\theta g}}{\partial r} + \frac{v_{\theta g}}{r} + f$ is the vertical component of absolute vorticity of

242 the gradient wind; and K = the turbulent diffusivity that is assumed to be a constant in this study.

243 The nonlinear Eqs. (17a) and (17b) could first be simplified using the scale analysis, and

244 then linearized as:

$$245 \quad \frac{v_{\theta g}}{r} \frac{\partial u'}{\partial \theta} - \xi_g v' = K \frac{\partial^2 u'}{\partial z^2} \quad (18a)$$

$$246 \quad \frac{v_{\theta g}}{r} \frac{\partial v'}{\partial \theta} + \xi_{ag} u' + \frac{v'}{r} \frac{\partial v_{\theta g}}{\partial \theta} = K \frac{\partial^2 v'}{\partial z^2} \quad (18b)$$

247 The analytical solution for this linear system is presented as follows (Snaiki and Wu 2016):

$$248 \quad u'(\theta, z') = (\alpha/\beta)^{1/2} \times Real \left\{ A_0 \times e^{(q_0 z')} + A_1 \times e^{(q_1 z' + i\theta)} + A_{-1} \times e^{(q_{-1} z' - i\theta)} \right\} = u_0 + u_1 + u_{-1} \quad (19a)$$

$$249 \quad v'(\theta, z') = Imag \left\{ A_0 \times e^{(q_0 z')} + A_1 \times e^{(q_1 z' + i\theta)} + A_{-1} \times e^{(q_{-1} z' - i\theta)} \right\} = v_0 + v_1 + v_{-1} \quad (19b)$$

250 where $\alpha = \frac{1}{2K} \xi_g$; $\beta = \frac{1}{2K} \xi_{ag}$; $q_1 = -(1+i) \left[\gamma + \sqrt{\alpha\beta} - \phi \right]^{1/2}$; $q_{-1} = -(1+i) \left[-\gamma + \sqrt{\alpha\beta} - \phi \right]^{1/2}$; $q_0 = -(\alpha\beta)^{1/4}$;

251 $\gamma = \frac{1}{2K} \frac{v_{\theta g}}{r}$; $\phi = \frac{1}{2Kr} \frac{\partial v_{\theta g}}{\partial \theta}$; and z' = new vertical coordinate used as the base of the computation scheme

252 where $z'=0$ is located above z_{10} (the 10 m height above the mean height of roughness elements)

253 (Meng et al. 1995). The other parameters are presented as follows:

$$254 \quad A_0 = \frac{-X_3}{X_1 + X_2 X_4} \quad (20a)$$

$$255 \quad A_1 = \frac{icC_d e^{-i\nu}}{4K(q_1 - q_{-1}^*)} (A_0 + A_0^*) \quad (20b)$$

$$256 \quad A_{-1} = \frac{icC_d e^{i\nu}}{4K(q_1^* - q_{-1})} (A_0 + A_0^*) \quad (20c)$$

$$257 \quad X_1 = \left[q_0 + \frac{fr}{K} C_d - \frac{2\eta C_d}{K} - \frac{c^2 C_d^2}{4K^2(q_1 - q_{-1}^*)} + \frac{c^2 C_d^2}{4K^2(q_1^* - q_{-1})} \right] \quad (20d)$$

$$258 \quad X_2 = \left[-q_0^* - \frac{fr}{K} C_d + \frac{2\eta C_d}{K} - \frac{c^2 C_d^2}{4K^2(q_1 - q_{-1}^*)} + \frac{c^2 C_d^2}{4K^2(q_1^* - q_{-1})} \right] \quad (20e)$$

$$259 \quad X_3 = -2 \frac{iC_d}{K} \left(\eta - \frac{fr}{2} \right)^2 \quad (20f)$$

$$260 \quad X_4 = - \left[-q_0 - \frac{fr}{2K} C_d + \frac{\eta C_d}{K} \right] \left/ \left[-q_0^* - \frac{fr}{2K} C_d + \frac{\eta C_d}{K} \right] \right. \quad (20g)$$

$$261 \quad \eta = \left[\frac{(-c \sin(\theta - \nu) - fr)^2}{4} + \frac{r}{\rho} \frac{\partial p}{\partial r} \right]^{1/2} \quad (20h)$$

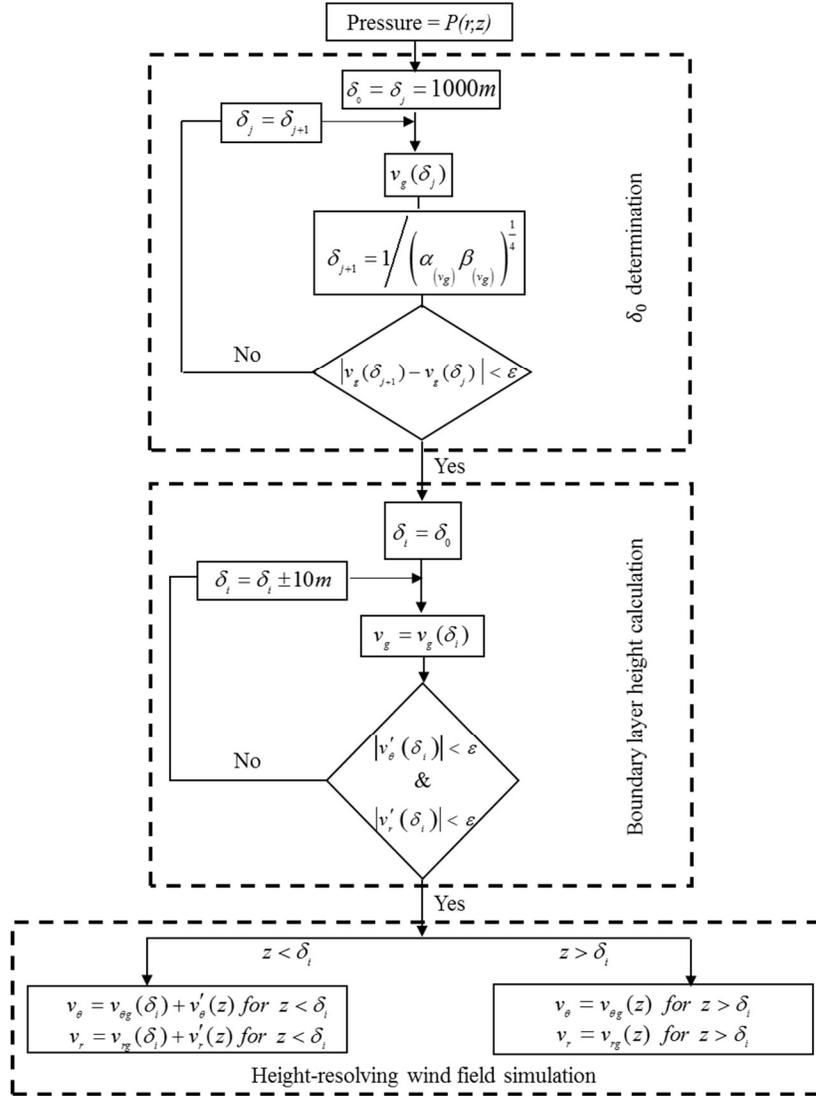
262 where C_d = drag coefficient; and (*) indicates a complex conjugate.

263 **3.3 An improved wind field simulation for tropical cyclone**

264 As mentioned in the preceding sections, the solution of the wind field could be conveniently
 265 obtained by prescribing a pressure field that is unchanged with height (e.g., Meng et al. 1996;
 266 Kepert 2001; Snaiki and Wu 2016). If the pressure variation with respect to the height is
 267 considered, various gradient wind values corresponding to different heights need to be calculated.
 268 This also leads to the mutual dependence of the gradient wind speed and the boundary layer depth.
 269 To obtain the accurate value of the gradient wind in the wind field simulation of the tropical
 270 cyclone, the iteration approach is utilized herein. In Snaiki and Wu (2016), the depth scale of the
 271 tropical cyclone was highlighted to give good estimate of the height where turbulent fluxes tend

272 to become negligible. Specifically, three depth scales of the tropical cyclone, namely $\delta_0 = 1/(\alpha\beta)^{\frac{1}{4}}$
 273 , $\delta_1 = 1/[\gamma + \sqrt{\alpha\beta} - \phi]^{\frac{1}{2}}$ and $\delta_{-1} = 1/[-\gamma + \sqrt{\alpha\beta} - \phi]^{\frac{1}{2}}$ corresponding to the frictional components
 274 (u_0, v_0) , (u_1, v_1) and (u_{-1}, v_{-1}) [Eq. (19)], respectively, have been defined. Since, (u_0, v_0) are the
 275 dominant frictional component, it is reasonable to select δ_0 as the initial value of the height of the
 276 boundary layer. A systematic way to calculate the height-resolving wind field in this study is
 277 illustrated in Fig. 7.

278 The first part of the flow chart of Fig. 7 is to determine the initial estimate of the boundary
 279 layer height δ_0 . Since δ_0 and the gradient wind speed depend on each other, the iteration process
 280 is necessary. Once the initial guess for the boundary layer height δ_0 is determined, the
 281 corresponding frictional wind speed components could be evaluated. The boundary layer height
 282 will be updated until the contribution from the friction become negligible. Based on the obtained
 283 boundary layer height δ_i , the wind field at certain height will be calculated using two different
 284 formulas. A constant value of the gradient wind speed evaluated at δ_i is utilized for the locations
 285 below the boundary layer height (i.e., $z < \delta_i$). Otherwise (i.e., $z > \delta_i$), the gradient wind speed is
 286 a function of height z , and the frictional components are equal to zero.



287

288

Fig. 7. Flow chart of wind field simulation methodology

289

A simple wind field simulation example is presented in Fig. 8, where $p_{c_0} = 940 \text{ hpa}$;

290

$r_m = 40 \text{ km}$; $B = 1.2$; $c = 10 \text{ m/s}$; $z_0 = 0.0001$; $k_m = 50 \text{ m}^2/\text{s}$; and $\nu = 90^\circ$, to highlight the

291

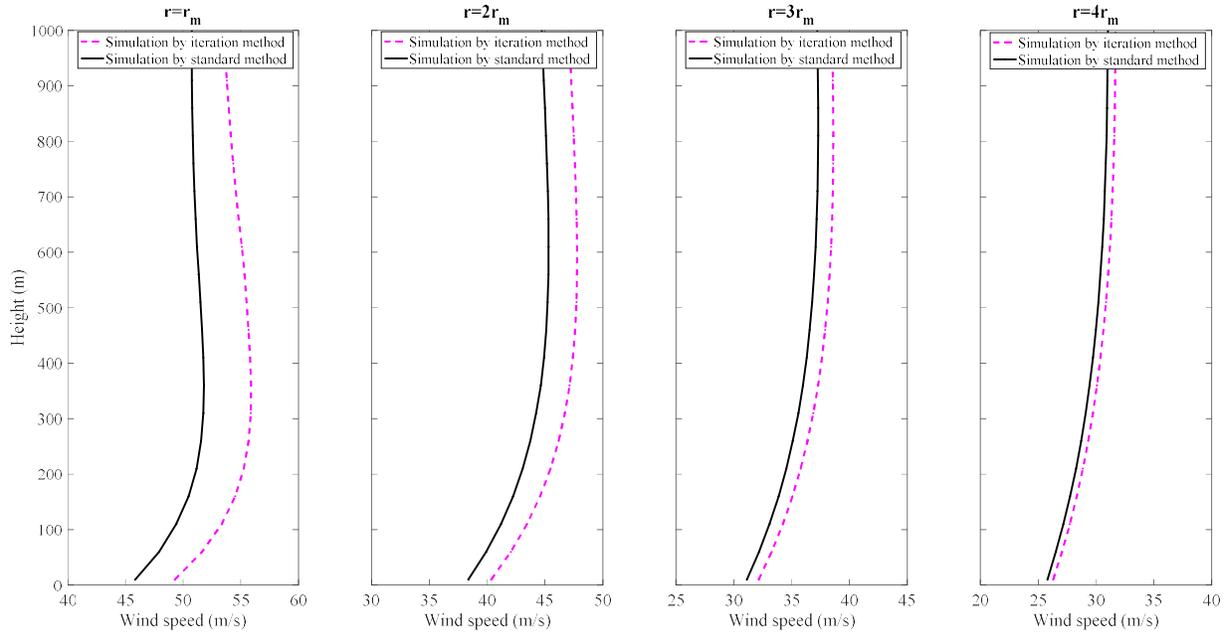
importance of considering the accurate gradient wind speed and boundary layer depth by iteration

292

method. The standard method indicates a constant value of the gradient wind speed is employed

293

in the wind field simulation.



294

295

Fig. 8. Simulation of the vertical wind profiles at different locations

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As shown in Fig. 8, the vertical wind profiles simulated using the iteration and standard schemes

297

present large difference. The difference becomes more significant as the location is close to the

298

radius of maximum wind and tends to decrease far away from the center of the tropical cyclone. It

299

should be mentioned that the tangential wind speed $v_{\theta g}$ in Huang et al. (2012) was considered to

300

vary with height from the ground surface rather than evaluated at the boundary layer depth, which

301

may need further improvement.

302

4. Model Validation

303

Wind records were obtained from the National Hurricane Center's North Atlantic Hurricane

304

Database (HURDAT). Typically, the parameters needed for the simulation are: ν approach angle;

305

c translation velocity of the hurricane; p_c central pressure; Δp central pressure difference; R_{\max}

306

radius of maximum winds; B Holland's parameter; ψ latitude; and λ longitude. The parameter

307

R_{\max} and B can be estimated using the methods available in the literature (e.g., Powell et al., 1991;

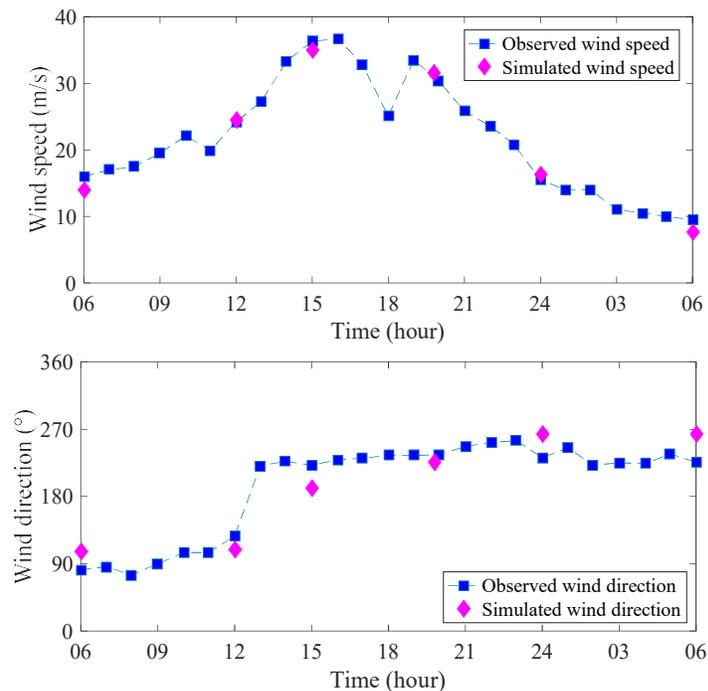
308 1998; Anthes 1982; Vickery et al., 2000b; Holland, 2008). In this study, the necessary information
309 is supplemented by the H*Wind snapshots. For the lapse rate parameters, the coefficients Γ_{r_m} ,
310 Γ_{eye} and Γ_0 can be approximated using the dropsonde data.

311 4.1 Surface wind simulation and validation

312 Two hurricanes, namely hurricane Bertha (1996) and Fran (1996) were selected for the surface
313 wind validation purpose. The 10 min averaged time was used for the observed wind data at 10 m
314 height for both hurricanes.

315 4.1.1 Hurricane Bertha

316 The anemometer is located on the FPSN7 station at (N33.44°, W77.74°). The parameters B and
317 R_{max} were found to be: $B = 1.2$ and $R_{max} = 70 km$. The observed wind speeds and directions were
318 compared with those obtained using the improved wind field model, and good agreement is
319 presented as in Fig. 9.



320

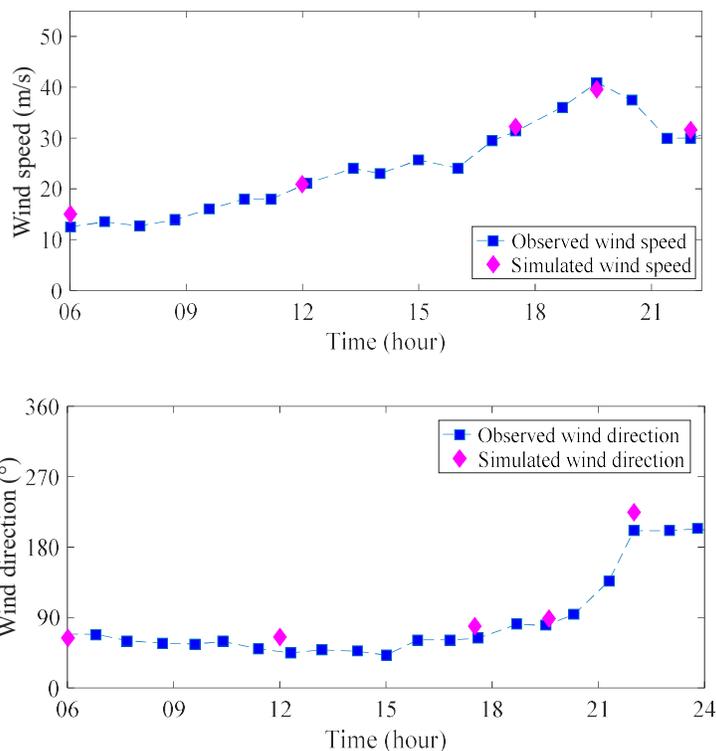
321

322

Fig. 9. Observed and simulated wind speeds (top) and directions (bottom) at FPSN 7, Hurricane Bertha

323 **4.1.2 Hurricane Fran**

324 The necessary parameters for the simulation were recorded by the marine station FPSN7 from
325 September 5th to September 6th. The station ID is 41013, located at (N33.44°, W77.74°). For
326 hurricane Fran $B=0.95$ and $R_{max} = 85 km$. As shown in Fig. 10, the results generated by the present
327 wind field model are consistent with hurricane Fran observations.



328

329

330 **Fig. 10.** Observed and simulated wind speeds (top) and directions (bottom) of Hurricane Fran

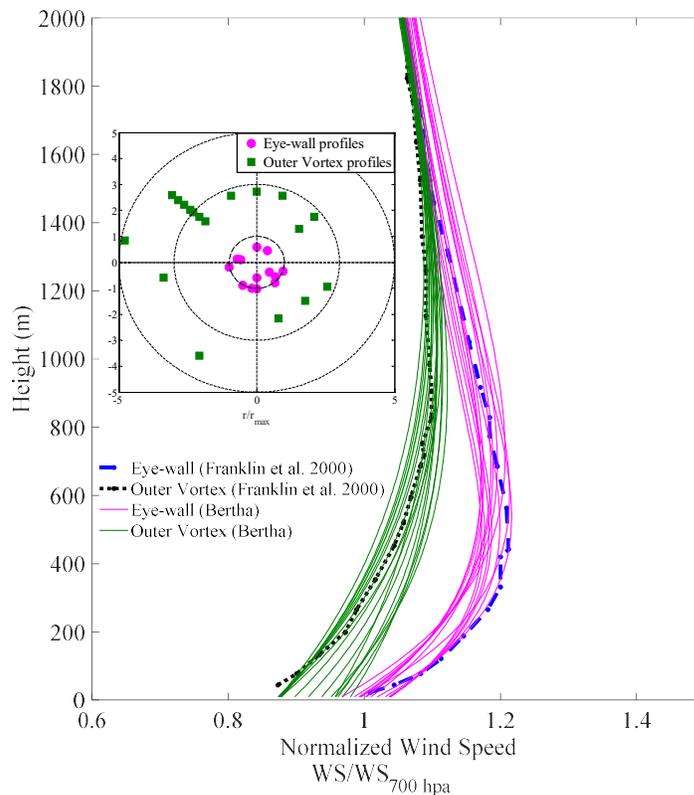
331 **4.2 Vertical wind profile simulation and validation**

332 Wind records from hurricane Bertha and Katrina were used to highlight the effects of the proposed
333 2-D pressure field on the tropical cyclone winds. Simulation vertical wind profiles of hurricane
334 Bertha in the eye-wall and outer-vortex regions were compared with the averaged wind profiles
335 obtained by Franklin et al. (2003). On the other hand, normalized wind profiles obtained by
336 dropsondes data were used to validate simulation vertical wind profiles of hurricane Katrina.

337 **4.2.1 Hurricane Bertha**

338 Comparison between mean wind speed profiles in the eye-wall and outer-vortex regions for a
339 specific hurricane is demonstrated to be very challenging. Franklin et al. (2003) constructed the
340 averaged wind profile based on numerous observations involving several hurricanes. By averaging
341 a large number of wind profiles from various hurricanes, good insight on the vertical profile of a
342 typical tropical cyclone could be obtained.

343 Several vertical wind profiles for hurricane Bertha at various locations of the eye-wall and
344 outer-vortex regions were constructed based on the improved wind field simulation. As shown in
345 Fig. 11, the simulation profiles present good agreement with the averaged one obtained by Franklin
346 et al. (2003) for both regions. Furthermore, it is noted that there is an obvious super-gradient region
347 for the eye-wall wind profiles (e.g., Kepert 2000; Kepert and Wang 2001; Snaiki and Wu 2016).



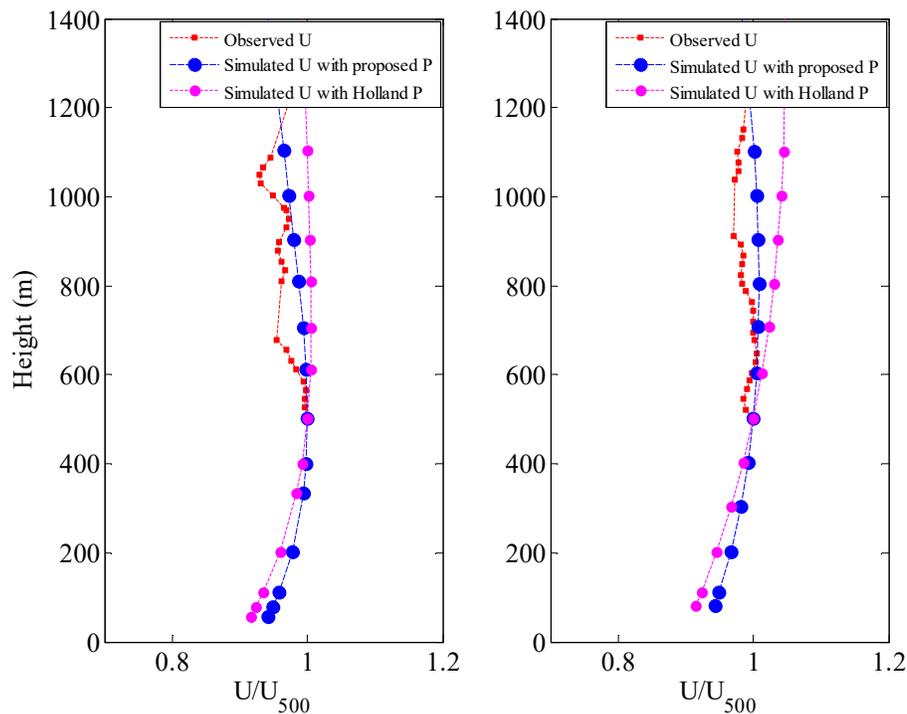
348

349

Fig. 11. Wind profiles in the eye-wall and outer vortex regions

350 **4.2.2 Hurricane Katrina**

351 Wind records from dropsondes (051926111) and (044535004) launched during hurricane Katrina
352 (2005) were used to validate the simulated wind profiles. In the comparison the wind profiles were
353 both normalized by a reference wind speed at 500 m. It should be noted that dropsondes only
354 provide the instantaneous wind speed profiles. Hence, more emphasis will be given to the high-
355 altitude comparison of observed and simulated results, where the mechanical turbulence is smaller.
356 To assess the effects of the proposed 2D pressure field on the wind profiles, the simulation results
357 based on the Holland's conventional pressure field are also presented. As indicated in Fig. 12, the
358 consideration of the proposed 2D pressure profile results in more accurate simulation of the wind
359 speeds.



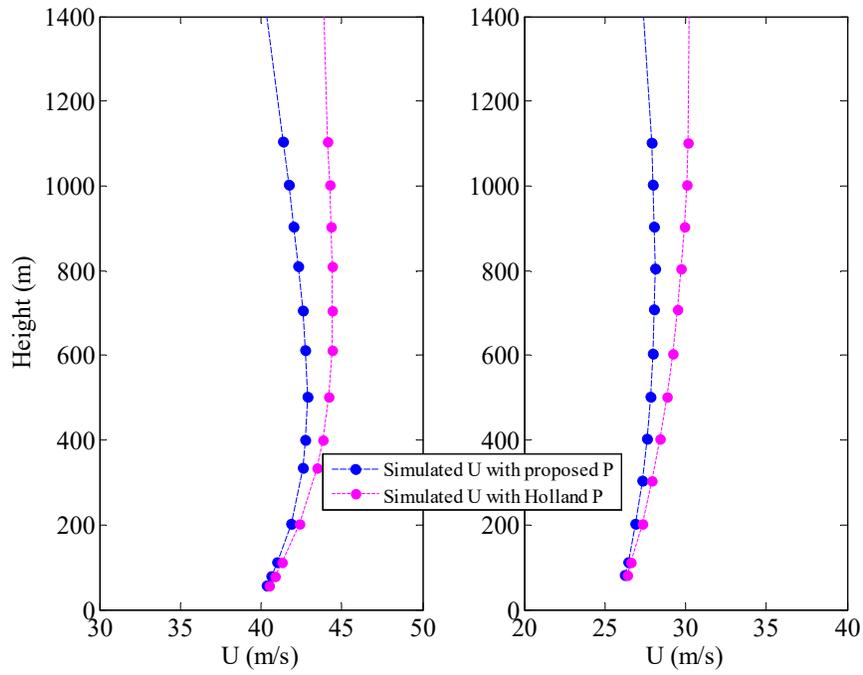
360

361 **Fig. 12.** Comparison of the normalized vertical wind profiles corresponding to two dropsondes during hurricane

362

Katrina (Left: 051926111 and Right: 044535004)

363 Furthermore, the simulated wind profiles plotted in Fig. 13 clearly present the significant
364 importance of an accurate pressure field on the wind field simulations.



365
366 **Fig. 13.** Comparison of the vertical wind profiles corresponding to two dropsondes during hurricane Katrina (Left:
367 051926111 and Right: 044535004)

368 5. Concluding remarks

369 A 2-D pressure model was proposed in this study, where the effects of temperature and moisture
370 were simultaneously accounted for through the virtual temperature. Furthermore, the linearized
371 consideration of the virtual temperature with respect to the height was introduced in the pressure
372 formula through the temperature lapse rate parameter. The empirical formulas constructed for
373 considering the spatial variation of the temperature lapse rate in the tropical cyclones greatly
374 simplified the simulations of pressure field. Then a framework based on the height-resolving
375 methodology was established to integrate the proposed 2-D pressure field into the boundary layer
376 wind field simulations of translating tropical cyclones. The improved wind field model involves

377 the iteration approach to systematically select an appropriate height for the calculation of gradient
378 wind speed, hence, it offers better simulation results that are more consistent with the tropical
379 cyclone observations. The improved height-resolving wind field simulations can be used in
380 conjunction with the Monte Carlo techniques to perform risk analysis of tropical cyclone hazards.
381 In addition, the present model also shows great promise in offering an improved method (based on
382 the proposed 2-D pressure field) to assess the climate change impacts on the wind field by
383 including some essential environmental parameters (e.g., temperature profile, moisture content).

384

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388

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