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Uniform distribution with MBO method of the infrared radiative energy in the thermoforming process of an ABS sheet

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This paper addresses the problem of distributing uniformly infrared radiative energy intercepted by a thermoplastic sheet surface during the infrared radiation transmitted by an oven with convection and conduction considerations. After discretizing this problem, we proposed an objective function that captures the uniform distribution of the radiative energy. With this approximation scheme, the corresponding problem appears to be nothing else than a variant of a quadratic assignment problem. Accordingly, we designed and applied a migrating bird optimization based algorithm (MBO for short), in order to minimize the corresponding objective function. To evaluate this approach we conducted a numerical experimental study.

Keywords: Thermoforming process, infrared radiative energy, convection and conduction, migrating bird optimization algorithm, quadratic assignment problem.

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1 Introduction

Polymers are invading more and more all the branches of industry to the point that they became essential for the modern industry. As a consequence, the performance of the processes that transform these materials into final products is a major step. The most used approach to form the final objects is the thermoforming process. This technique uses a heating phase to deform the thermoplastic materials according to a predefined mold (the forming step), and then a cooling phase to make these material retrieve their initial physical properties while keeping the form they got at the end of the previous phase. The quality of the formed object depends strongly on the thermal process imposed on the surface of the thermoforming sheet during the heating phase [9]. Indeed, the distribution of the thicknesses of the object depends essentially on the temperatures assigned to the oven heating elements [5].

When heated by infrared radiative energy (IR-energy), the plastic sheet is transformed from a glassy state into a rubbery state. This hot state combined with the gravity creates a non-uniform thickness distribution in the plastic sheet. Adequate optimization of the heating stage can improve significantly the mass distribution in the finished part. An effective way to achieve better uniform thickness distribution is to reduce the differences of energy intercepted and absorbed by the different areas of the thermoplastic sheet. An illustration is pictured by Figure 1, in which the colors represent the intensity of energy received by the thermoforming sheet.

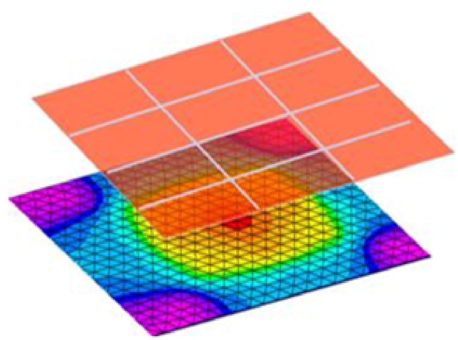


Figure 1: Heating stage: thermoforming process

The main goal of this paper is to determine, from a given set of temperatures, the best distribution of the temperatures assigned to the oven heating elements in order to minimize the difference of the temperature between the different areas the thermoplastic sheet.

The present paper is organized as follows. Section 2 introduces the thermal model we are addressing. In Section 3 we present the problem we are considering and the corresponding optimization formulation we derived. In Section 4, we present the migrating bird optimization metaheuristic method we used to solve the optimization model. Section 5 first presents details on the implementation issues of the MBO algorithm, and then the results of the experimental study we conducted on the quality of the solutions produced by our solution. Concluding remarks are presented in Section 6.

2 Thermal model of the oven and the thermoplastic sheet

The exchanges of energy between the oven and the thermoforming surface of the sheet are supposed to be of radiative type. The heating of the sheet is undertaken on its two sides through the lower and the upper heating elements of the oven. The oven has no system of ventilation, and the cooling is made through natural convection between the sides of surfaces of the sheet and the ambient air. The convection between the heater and outside is neglected. The sheet temperature is very low in comparison to the temperatures of the heating elements. The cold material hypothesis is therefore justified here.

In this study we used an ABS (Acrylonitrile-Butadiene-Styrene) opaque sheet. The volume heat absorption term is considered to be negligible. The thermal parameters of the sheet are considered constant, and its thickness is very thin. Consequently, the heat transfer by conduction is in the vertical direction of the thickness of the sheet [8]. The equation which governs the distribution of the temperature in the thickness of the sheet is as follows:

$$\rho c_p \frac{\partial T}{\partial t} = k \frac{\partial^2 T}{\partial z^2}, \tag{1}$$

where ρ is the density, c_p the specific heat capacity, T the temperature, t the time, and k the conductivity of the thermoplastic sheet, which we assume to be homogeneous and isotropic. In this context, the thermal properties were fixed to the average values.

The thermoplastic sheet receives heat through radiation and convection on its top and bottom surfaces. Hence, the boundary conditions of Equation 1 considered in this study are

$$k \frac{\partial T}{\partial z}.n + h(T_s - T_\infty) + Q.n = 0,$$

with T_s denotes the temperature of the thermoplastic sheet, T_∞ the temperature of ambient air, and h the convection coefficient. The convection heat term, $h(T_s - T_\infty)$, represents the transfer from the surface sheet to the environment, whereas Q is the IR-energy, and n is the outward normal to the surface.

Equation 1 is solved numerically : the thermoplastic sheet is subdivided into $n = n_1 \times n_2$ rectangular surface areas $S_{(i,j)}, i = 1, \ldots, n_1; j = 1, \ldots, n_2$, arranged in a matrix of dimension $(n_1 \times n_2)$. On the other hand, we assume that the oven has a set of $(m = m_1 \times m_2)$ heating elements on the upper and lower levels, respectively (see Figure 2). Each element consists of a ceramic flat surface $S_{(e,\ell)}, e = 1, \ldots, m_1; \ell = 1, \ldots, m_2$, receiving temperatures from set $\tau = \{\tau_1, \tau_2, \cdots, \tau_p\}$.

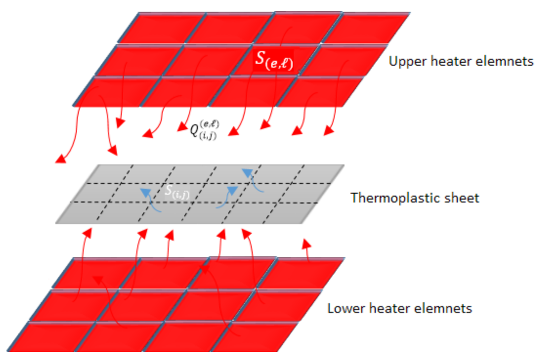


Figure 2: Sheet and oven discretization

The IR-energy, Q, received by the surface of the thermoplastic material governs the present study. The IR-energy on cell (i,j) of the thermoplastic sheet of surface $S_{(i,j)}$ is calculated according to the geometric orientation and temperature of oven heater cell (e,ℓ). View factors are required to obtain the energy for each cell of the thermoplastic surface exposed to the IR-energy. We assume that surface $S_{(e,\ell)}$ of the heater is separated by a transparent medium, which is semi-transparent, from surface $S_{(i,j)}$ of the thermoplastic sheet. As a consequence, after some approximation manipulations, the amount of IR-energy $Q^{(e,\ell)}_{(i,j)}$ leaving surface $S_{(e,\ell)}$ and intercepted by surface $S_{(i,j)}$ for a given temperature $T_{(e,\ell)}$ is approximated as follows [6]:

$$Q^{(e,\ell)}_{(i,j)} = F^{(e,\ell)}_{(i,j)} \frac{S_{(e,\ell)}}{S_{(i,j)}} \bar{\varepsilon} \sigma T^4_{(e,\ell)}, \tag{2}$$

where $F^{(e,\ell)}_{(i,j)}$ denotes the view factor that calculates the fraction of energy emitted by cell $S_{(e,\ell)}$ and received by cell $S_{(i,j)}$, $\bar{\epsilon}$ the average source emissivity of the material. Parameter σ is the Stefan-Boltzman constant of value $5.67 \times 10^{-8}\ W.m^{-2}.K^{-4}$, and finally $T_{(e,\ell)}$ is the temperature assigned to cell (e,ℓ). The view factors are given by the following:

$$F^{(e,\ell)}_{(i,j)} = \frac{1}{S_{(e,\ell)}} \int_{S_{(e,\ell)}} \int_{S_{(i,j)}} \frac{\cos\theta_{(e,\ell)} \cos\theta_{(i,j)}}{\pi \times r^2} dS_{(i,j)} dS_{(e,\ell)}.$$

Parameter r is the distance separating cells (i, j) and (e, ℓ), and $dS_{(i,j)}$ and $dS_{(e,\ell)}$ are the elemental areas connected by line r, which forms the polar angles $\theta_{(i,j)}$ and $\theta_{(e,\ell)}$ with the surface normal $n_{(i,j)}$ and $n_{(e,\ell)}$, respectively (see Figure 3).

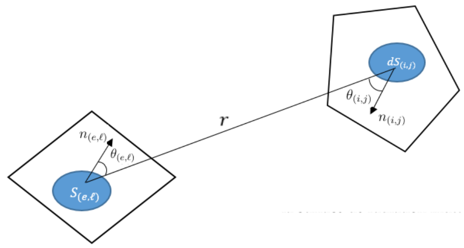


Figure 3: View factor

3 Statement of the optimization problem

The quality of the molded product depends heavily on the distribution of the temperatures generated by the energy flux received by the surface areas of the thermoplastic sheet exposed to the IR-energy. More precisely, this energy spreading relies on the heating arrangement of the zones of the oven. Adequate modeling and optimization of the heating stage can improve significantly the final thickness distribution in the final product, thus improving its quality and reducing the number of rejections, which in turn improves productivity [7]. Let us recall that the energy received by each cell of the thermoplastic sheet from a single heating cell of the oven is expressed by Equation (2). The idea of minimizing the IR-energy gap received by the thermoplastic sheet areas may be constructed as follows. The difference in IR-energy between the areas of the medium and those of the edges of the thermoplastic sheet plastic sheet surface has an important influence on the quality of the final shape of the product. Obviously, the smaller are the gaps between these elements the better is the quality of the product. The goal is thus to make that the elements of the thermoplastic sheet receive approximately the same amount of IR-energy from the heating elements. One way to capture this goal is to minimize the standard deviation of the energy received by the cells of the thermoplastic sheet. This process leads to the following optimization problem.

Given are m heating cells of an oven arranged as a (m_1, m_2)-matrix with $m = m_1 \times m_2$. A temperature from set $\tau = \{\tau_1, \ldots, \tau_q\}$, the temperature set, is assigned to each heating cell of the oven in order to minimize the standard deviation of the set of fractions of the energy received by the n thermoplastic sheet cells, also arranged as a (n_1, n_2)-matrix with $n = n_1 \times n_2$. This problem is a variant of the quadratic assignment problem, known to be $\mathcal{NP}$-hard problem is the strong sense [3].
Let $x^k_{(e,\ell)}$ be a decision variable such that $x^k_{(e,\ell)} = 1$ if temperature $\tau_k \in \tau$ is assigned to cell (e, ℓ), and 0 otherwise. Then, for the τ_k temperature received by the element heater (e, ℓ), the energy received by cell (i, j) is the follows :

$$Q^{e,\ell,k}_{i,j} = F^{(e,\ell)}_{(i,j)} \frac{S_{(e,\ell)}}{S_{(i,j)}} \bar{\varepsilon}\sigma\tau_k^4 \times x^k_{e,\ell}.$$

The total energy received by cell (i, j) of the thermoplastic sheet is

$$Q_{i,j} = \sum_{e=1}^{m_1} \sum_{\ell=1}^{m_2} Q^{e,\ell,k}_{i,j}, \quad k \in \{1, \ldots, p\}.$$

If we denote by $\overline{q}$ the average of the energy received by the $n = n_1 \times n_2$ thermoplastic sheet cells, that is

$$\overline{q} = \frac{1}{n} \sum_{i=1}^{n_1} \sum_{j=1}^{n_2} Q_{i,j}.$$

The objective function we are addressing is therefore expressed as follows:

$$\min f(Q_{i,j}) = \frac{1}{\overline{q}}\sqrt{\frac{1}{n}\sum_{i=1}^{n_1}\sum_{j=1}^{n_2}(Q_{i,j}-\overline{q})^2}, \tag{3}$$

Let us note that if the standard deviation was not divided by $\overline{q}$, then the minimization process would favor the small values of the temperature set as they produce small values for that objective function.

The above objective function is subject to the following constraints: a heating cell must receive exactly one temperature from set τ, a temperature from set τ is used at most $(n_1 \times n_2)$ times, and the temperature τ_k of an element heating (e, ℓ) is strictly chosen from the set $\tau = \{\tau_1, \tau_2, \cdots, \tau_p\}$. These constraints are respectively expressed as below:

$$\begin{aligned} &\sum_{k=1}^{q} x_{e,\ell}^{k} = 1, && e = 1,\ldots,m_1, \ell = 1,\ldots,m_2,\\ &\sum_{e=1}^{m_1}\sum_{\ell=1}^{m_2} x_{e,\ell}^{k} \le m_1 \times m_2, && k = 1,\ldots,p,\\ &\tau_k \in \tau = \{\tau_1,\tau_2,\cdots,\tau_p\}, && k = 1,\ldots,p. \end{aligned}$$

In order to get the thermoplastic sheet ready for the thermoforming, the corresponding temperature, which corresponds to IR-energy, has to be within its thermoforming window. This constraint is expressed as follows:

$$q_{\min} \le Q_{i,j} \le q_{\max}, \quad i = 1,\ldots,n_1, \quad j = 1,\ldots,n_2.$$

4 Migrating bird optimization method

The Migration Birds Optimization (MBO) was designed by Duman et al. [4]. This metaheuristic method is inspired from the 'V'-shape of the flights of migrating birds. The property of bird flights lies in the energy conservation. Indeed, when a bird beats its wings, it generates a draft which will make the birds behind have to supply fewer efforts to rise. The organization of the flight of the birds is as follows: the bird in the head leads the group for a certain period of time, and spends more energy than the rest of its congeners. When it is tired, it moves behind the line of the group, and one of the birds currently behind takes the lead. The parameters defining the above metaheuristic algorithm are p: the number of initial solutions, α: the number of neighbor solutions to consider, β: the number of neighbors to share with the next solution, γ: the number of iterations to perform before changing the leader solution, and L: the maximum number of iterations the algorithm executes. In [2], through their experimental study they conducted, the value of these parameters are: $p = 51$, $\alpha = 7$, $\beta = 3$, $\gamma = 10$ and $L = 10\ 000$. The MBO algorithm may be resumed as in Procedure 1.

Algorithm 1 MBO

```
Fix p, α, β, γ and L;
Generate at random p solutions, and place them in a V-shape;
for (i=0; i < L; i++) do
  for (j=0; j < γ; j++) do
    Improve the leader solution by generating and evaluating α of its neighbors and i ⟵ i + α;
    Except the solution leader, improve the solution in the V-shape by evaluating (α − β) neighbors with the β best solutions
    not used in the solution ahead and set i ⟵ i + (α − β);
  end for
  Move the leader to the back of the group, and move one of the next solutions that are behind it to the leader position;
end for
Return the best solution;
```

In order to make the above algorithm operational, the neighborhood of a solution must be specified [1]. The one we adopted is as in Procedure 2.

Algorithm 2 Neighborhood

Generate randomly a number $r (0 \leq r \leq 1)$;
if $(r \leq 0.5)$ **then**
  Select at random two temperature locations i and j from the actual solution;
  Exchange the two corresponding temperatures.
else
  Select at random a temperature location i.
  Choose at random a temperature from set τ and assign it to location i.
end if

5 Experimental study

To make our numerical simulation closer to real world applications, the dimensions of the thermoforming oven has an industrial scale. The oven is made of 144 (12×12) elements from above and below parts, each of is of dimension 0.06m by $0.120m$ and their emissivity is 0.85. The lateral sides of the oven are open and the environment behaves as a black body and The temperature for each element is between 300 K to 900 K with a step of 5. The thermoplastic sheet ABS is heated until reaching the thermoforming window $140°C$- $160°C$. The thermoplastic sheet, of dimension $1.95m \times 1.45m \times 0.012m$, is placed at 0.20m from the upper and the lower parts of the oven. The heating time is fixed to 90 seconds. The thermal properties of the thermoplastic sheet are considered as independent from the temperatures. The average values that we considered are summarized as below [9]:

Thermal properties of ABS	Values
Specific heat $(J/kg.k)$	2590
Density $(kg.m^{-3})$	1050
Thermal conductivity $(W/m.k)$	0.174
Initial temperature $(°C)$	$40°C$

5.1 Analysis of the numerical results

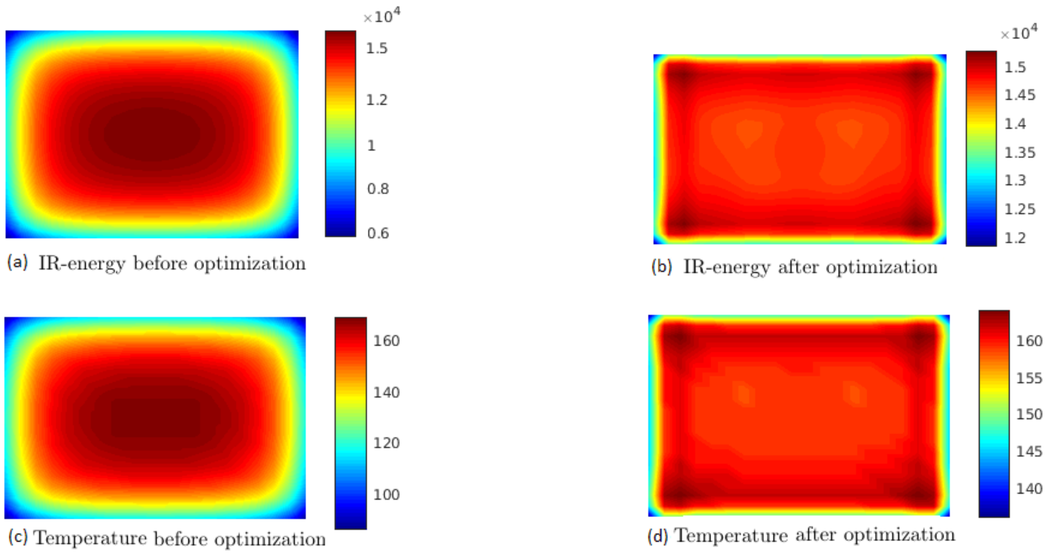
The MBO metaheuristic was coded in C++ language, and debugged using Microsoft Visual Studio 2015 on an hp machine with an Intel(R)Core(TM) i5-6200U processor and a RAM of 16Go.

The first stage of the simulation is dedicated to search the solution with the MBO approach. This algorithm was tested on 10 instances generated randomly. The solution that produces the best value of our objective function is selected. The results are summarized in Table 1. Let us note that these values, generated by our meta-heuristic MBO algorithm, correspond to the temperature of each of the (12×12) heating elements of the oven.

Table 1: Heater temperatures of the upper part of the oven

i, j	1	2	3	4	5	6	7	8	9	10	11	12
1	890	715	760	755	750	755	755	750	755	760	715	890
2	580	455	510	470	475	480	480	475	470	510	455	585
3	740	560	620	605	620	615	615	620	605	620	560	740
4	685	540	595	575	565	565	565	565	575	595	540	685
5	705	540	595	580	580	590	590	580	580	595	540	705
6	700	545	595	580	580	585	585	580	580	595	545	700
7	700	545	595	580	580	585	585	580	580	595	545	700
8	705	540	595	580	580	590	590	580	580	595	540	705
9	685	540	595	575	565	565	565	565	575	595	540	685
10	740	560	620	605	620	615	615	620	605	620	560	740
11	580	455	510	470	475	480	480	475	470	510	455	585
12	890	715	760	755	750	755	755	750	755	760	715	890

To show the quality of the solution produced by the MBO metaheuristic, we made a comparison between the radiative heat flux distribution obtained with the MBO approach of Table 1 and that calculated with a solution of which all the oven heating elements are fixed to $600K$. Figure 4 illustrates the flow and temperature distributions on the surface of the thermoplastic sheet before and after optimization. Figures 4a and 4b summarize those results, respectively. We may observe that before optimization, the radiative flux gap is (9×10^3 W.m^{-2}) for a maximum of (15×10^3 W.m^{-2}) concentrated in the middle and a minimum of ($6 \times (10^3$ W.m^{-2}) located at the borders of the surface of the thermoplastic sheet. For the same heat flow profile, after optimization, the difference is reduced by (3×10^3 W.m^{-2}). The edges of the thermoforming plastic sheet surface are more heated and the heat flux distribution is much better for a maximum of (15×10^3 W.m^{-2}) and a minimum of (12×10^3 W.m^{-2}).



(a) IR-energy before optimization
(b) IR-energy after optimization
(c) Temperature before optimization
(d) Temperature after optimization

Figure 4: Distribution of the IR-energy and the temperature of the thermoplastic sheet

Figures 4c and 4d illustrate the distribution of the temperatures on the surface of the sheet before and after optimization of the oven temperatures, respectively. The results show that the temperature difference between the center and the the borders of the sheet is greater before optimization. Indeed, for uniform heating of the temperatures of the elements of the oven, the temperature gap at the surface of the sheet is $60°C$ for a maximum of $160°C$ concentrated in the middle and a minimum of $100°C$ located at the borders of the surface of the thermoplastic sheet. With optimized heating, the gap temperatures between the borders and the center of the sheet is reduced to $20°C$. Indeed, the borders of the sheet are better heated with a temperature of $140°C$ and a central zone of $160°C$. Moreover, in the case of non-optimized heating, the temperatures of the sheet borders do not reach the minimum forming temperature of $140°C$. It may be a cause of sheet forming defects. When using MBO approach, the sheet temperature is in the forming range for all regions of the thermoforming sheet.

Figures 5 show the value improvement of the best solution versus the number of evaluated solutions. The results show, after 6500 iterations, the MBO method converged to the best solution. The execution time of the method does not exceed 23 minutes.

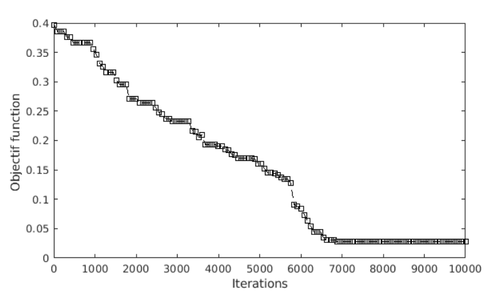


Figure 5: Improvement of the best solution

6 conclusion

We addressed in this paper the problem of distributing uniformly infrared radiative energy intercepted by a thermoplastic sheet surface during the infrared radiation transmitted by an oven with convection and conduction considerations. After discretizing this problem, we proposed an objective function that captures the uniform distribution of the radiative energy. With this approximation scheme, the corresponding problem appears that it nothing else than a variant of a quadratic assignment problem. Then, we designed and applied a migrating bird optimization based algorithm (MBO for short), in order to minimize the corresponding objective function.

We then conducted an experimental study to evaluate the quality of the solution produced by the MBO algorithm. This study reveals that the solution generated by MBO improves the distribution of temperature and the infrared radiative energy intercepted by the thermoplastic sheet surface during the infrared radiation stage.

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